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A MODEL FOR GRAIN MISALIGNMENT IN CYLINDRICAL PORT MOTORS

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This report presents a mathematical model of the geometry of the propellant grain of a cylindrical port motor cast with a misaligned mandrel. Also presented is an HP-41C calculator program which incorporates this model, and an example demonstrating the application of the misaligned motor geometry.
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## I. INTRODUCTION

The purpose of this report is to present a mathemetical model of the geometry of a cylindrical port motor cast with a misaligned maidrel. This model was developed to determine the burning surface area and free volume of such motors.

This report also includes a detailed description of the geometry model. In formulating this model, two basic types of mandrel misalignment were considered: mandrel displacement and mandrel cocking. In addition to the model description, two appendices are included. Appendix A presents an HP-41C calculator program and Appendix B presents an example of the application of the geometric model.

The details presented in this report are the result of work conducted at the Propulsion Directorate of the US Army Missile Command. The purpose of this work was to obtain a better insight into the geometrical nature of mandrel misalignment.

## II. GENERAL

The cylindrical port grain is one of the most versatile and widely used solid rocket motor configurations. This motor geometry is widely employed throughout the industry. One of the more common applications of cylindrical port grains is in subscale ballistic test motors. The characterization of propellant burning rates is one of the primary uses of the subscale test motors. Typically, when a cylindrical port motor is employed in burning rate characterization, the motor is designed with a burning surface area profile that is essentially constant with respect to web distance burned. Thus, when fired, the motor will operate at a relatively constant chamber pressure. The burning rate of the propellant, at the average operating pressure of the motor, is determined by dividing the web thickness of the motor by the burn time. This entire analysis method is based on the assumption that the web thickness of the motor is a known quantity. Therefore, it is essential to this method that the web distance be uniform over the entire length of the grain. As a result of this assumption, a major source of experimental error in the determination of burning rate from ballistic test motor firings is ballistic test motors that do not have a uniform web.

The major cause of variations in the web thickness for cylindrical port motors is mandrel misalignment. Mandrel misalignment essentially means that when the motor was cast the axis of symmetry of the mandrel (and thus of the motor port) did not colncide with the axis of symmetry of the motor case. This condition causes a variation of the web thickness over the length of the grain which means that the burning surface will not contact the motor case wall uniformly. As a result, the burning rate analysis method which is based on the assumption that the entire burning surface contacts the motor case wall at the same instant and is rendered useless.

Since the cylindrical port motor is such a basic propellant development tool, it is essential to obtain a better understanding of the influences of mandrel misalignment on the performance of such motors. The first step in obtaining this understanding is to acquire a knowledge of the geometry of misaligned motors. It should be noted that the effects of mandrel misalignment on the performance of solid rockets were extensively investigated by Maykut [1]. The purpose of these studies was to investigate the effact of various grain asymmetries on the delivered impulse of a rocket motor. In these studies a generalized grain geometry computer code was employed. One feature of this code was the ability to solve for the surface histories of various asymmetric propellant grains [2]. While this code was capable of analyzing the geometry of a misaligned cylindrical port motor, the general nature of the code made it somewhat cumbersome to use. As a result, it was considered advantageous to independently develop a geometry model for the specific class of motors considered in this study.

## III. MANDREL MISALIGNMENT

The first step in considering mandrel misalignment in a cylindrical port rocket motor is to consider the general geometry of the motor. In a perfectly aligned motor, the port of the grain and the motor case will have the same axis of symmetry. Figure 1 shows the geometry of such a motor. The problem created by mandrel misalignment is that the motor port and motor case do not have a common axis of symmetry. In order to begin evaluation of the nature of mandrel misalignment, first consider the case where the port and case axes are parallel but do not coincide. A cross-section of the motor taken through a plane perpendicular to the axes of symmetry will reveal circular port and motor case crosssections. These circles are not, however, concentric. As the propellant port burns out radially the radius of the port will increase. Eventially one point on the port cross-section will contact the case wall. This point defines the region where the misaligned motor differs from the perfectly aligned motor. Until the point of contact the aligned and misaligned motors will exhibit the same burning surface area history.

For the aligned motor, wall contact occurs along the entire periphery and thus indicates the time of motor burnout, while for the misaligned motor, wall contact creates a sliver zone. This sliver zone is the cross-sectional area of propellant remaining at the point of first wall contact. The misaligned motor will continue to operate as the sliver zone burns out. This sliver zone has a surface area that decreases as web distance burned increases. The sliver will result in an extended motor tail-off on the pressure-time trace for the misaligned motor. Figure 2 presents the burning profile for a misaligned crosssection.

The next step is to develop a mathematical model of the misaligned crosssection. Consider the misaligned port for the propellant grain at a given cross-section:

The radius of the propellant grain is given by:

$$
\begin{equation*}
R(\tau)=R(0)+\tau \tag{1}
\end{equation*}
$$



Figure 1. Cross-section of cylindrical port motor (perfectly alined).


Figure 2. $\frac{\text { Cross-section burn profile for C-P grain cast with }}{\text { misaligned mandrel. }}$

Where:
$R(\tau)$ is the radius of the grain
$R(0)$ is the initial grain radius
and $\tau$ is the web distance burned.
The intersection of the propellant port and the motor case is given by the coordinates:

$$
\left(X_{I}, \pm Y_{I}\right)
$$

If
$R(\tau)<R_{C}-\Delta X$

There is no intersection
If

$$
\begin{align*}
& R(\tau) \geq R_{C}-\Delta X \\
& X_{I}=\frac{R^{2}(\tau)-R_{C}^{2}-\Delta X^{2}}{2 \Delta X} \tag{2}
\end{align*}
$$

$Y_{I}=\sqrt{R_{C}^{2}-X_{I}^{2}}$
Where
$\mathrm{R}_{\mathrm{C}}$ is the inside radius of the motor case
$\Delta X$ is the magnitude of the mandrel offset
$X_{I}$ is the $X$-coordinate of the intersection and
$Y_{I}$ is the $Y$-coordinate of the intersection.
The perimeter of the burning surface of propellant at a given crosssectional plane is given by:

$$
\begin{equation*}
P(\tau)=\frac{\pi}{180} \quad \theta R(\tau) \tag{4}
\end{equation*}
$$

Where:
If

$$
\begin{align*}
& R(\tau) \leq R_{C}-\Delta X \\
& \theta_{1}=360^{\circ} \tag{5}
\end{align*}
$$

If

$$
\begin{align*}
& X_{I}<-\Delta X \\
& \theta_{1}=360^{\circ}-2 \operatorname{Tan}^{-1} \frac{Y_{I}}{-\Delta X-X_{I}} \tag{6}
\end{align*}
$$

If

$$
\begin{align*}
X_{I} & =-\Delta X \\
\theta_{1} & =180^{\circ} \tag{7}
\end{align*}
$$

If

$$
\begin{align*}
& X_{I}>-\Delta X \\
& \theta_{1}=2 \mathrm{TAN}^{-1} \quad \frac{Y_{I}}{X_{I}+\Delta X} \tag{8}
\end{align*}
$$

## Where:

$$
\mathrm{P}(\tau) \text { is the perimeter of the propellant. }
$$

The cross-sectional area of propellant at a given cross-sectional plane is given by:

$$
\begin{equation*}
A_{c r}(\tau)=\frac{\pi}{360}\left(R_{c}^{2} \theta_{2}-R^{2}(\tau) \theta_{1}\right)+2 A_{1} \tag{9}
\end{equation*}
$$

Where
If

$$
\begin{align*}
& \mathrm{R}(\tau) \leq \mathrm{R}_{\mathrm{c}}-\Delta \mathrm{X} \\
& \theta_{2}=360^{\circ}  \tag{10}\\
& \mathrm{A}_{1}=0 \tag{11}
\end{align*}
$$

If

$$
\begin{align*}
& \mathrm{X}_{\mathrm{I}}<0 \\
& \theta_{2}=360^{\circ}-2 \mathrm{TAN}^{-1} \frac{\mathrm{Y}_{I}}{-\mathrm{X}_{\mathrm{I}}} \tag{12}
\end{align*}
$$

If

$$
\begin{align*}
& \mathrm{X}_{\mathrm{I}}=0 \\
& \theta_{2}=180^{\circ}  \tag{13}\\
& \text { If }
\end{align*}
$$

$$
\begin{align*}
& \mathrm{X}_{\mathrm{I}}>0 \\
& \theta_{2}=2 \operatorname{TAN}^{-1} \quad \frac{\mathrm{Y}_{\mathrm{I}}}{\mathrm{X}_{\mathrm{I}}} \tag{14}
\end{align*}
$$

And

$$
\begin{equation*}
A_{1}=\left[S(S-\Delta X)(S-R(\tau))\left(S-R_{c}\right)\right]^{1 / 2} \tag{15}
\end{equation*}
$$

## Where:

$$
\begin{equation*}
S=1 / 2\left(\Delta X+R(\tau)+R_{C}\right) \tag{16}
\end{equation*}
$$

Where:
$A_{C r}(\tau)$ is the propellant cross-sectional area
At a cross-sectional plane, the distance for the shortest propellant web is given by:

$$
\begin{equation*}
\tau_{s w}=R_{c}-R(0)-\Delta X \tag{17}
\end{equation*}
$$

The web distance for total propellant burnout at a cross-section is given by:

$$
\begin{equation*}
\tau_{\mathrm{pbo}}=\mathrm{R}_{\mathrm{c}}-\mathrm{R}(0)+\Delta \mathrm{X} \tag{18}
\end{equation*}
$$

A complete cross-sectional view of the propellant grain is shown in Figure 3.


Figure 3. Cross-sectional view of C-P grain cast with an offset mandrel.

With the cross-sectional geometry of the propellant grain completely detailed, the next step is to consider the geometry of the entire motor. In order to consider the motor geometry a set of coordinate systems must be established. Two coordinate systems will be considered, one for the motor case and one for the mandrel. Descriptions of the coordinate systems are as follows:

For the motor case -
X - An axis in a plane perpendicular to the axis of symmetry of the motor case
$Y$ - An axis in the same plan as the $X$-axis and perpendicular to the $X$ axis and the axis of symmetry of the motor case

Z - The axis of symmetry of the motor case.
For the mandrel -
$\bar{X}$ - An axis in a plane perpendicular to the axis of symmetry of the mandrel
$\bar{Y}-A n$ axis in the same plane as the $\bar{X}$-axis and perpendicular to the $\bar{X}$ axis and the axis of symmetry of the mandrel
$\overline{\mathrm{Z}}$ - The axis of symmetry of the mandrel.
Two possible cases of mandrel misalignment will be considered. The first case is a displaced mandrel and the second is a cocked mandrel. The following are descriptions of the two resulting motor geometries.
A. Displaced Mandrel

In the case of the displaced mandrel, the assumption is made that the sides of the mandrel are parallel to walls of the motor case but the axis of symmetry of the mandrel (Z-axis) is displaced a distance $\Delta X$ from the axis of symmetry of the motor_case ( Z -axis). Thus, the $X$ and $\bar{X}$ axes are colinear, the $Y$ and $\bar{Y}$, and the $Z$ and $\bar{Z}$ axes, respectively, are parallel. The geometry is presented in Figure 4.


Figure 4. Displaced mandrel configuration.

For the displaced mandrel the propellant cross-section at each $Z$ coordinate is the same. Therefore, for a given web distance burned the propellant perimeter and cross-sectional area are constant with respect to 2. Thus, the propellant burning surface area is given by:

$$
\begin{equation*}
A_{b}(\tau)=L(\tau) P(\tau)+A_{c r}(\tau) N_{e b} \tag{19}
\end{equation*}
$$

Where
R ( $\tau$ ) is given by Equation (1)

$$
\begin{equation*}
\text { and } \quad \mathrm{L}(\tau)=\mathrm{L}(0)-2 \tau \mathrm{~N}_{\mathrm{eb}} \tag{20}
\end{equation*}
$$

Where
$A_{b}(\tau)$ is the burning surface area of the motor
$L(\tau)$ is the length of the grain
L (0) is the initial length of the grain and
$N_{\text {eb }}$ is the number of ends that are burning.
The free volume of the motor is given by:
$V(\tau)=\pi R_{c}{ }^{2} L(0)-L(\tau) A_{c r}(\tau)$

Where
$V(\tau)$ is the free volume of the motor.
B. Cocked Mandrel

In the case of the cocked mandrel two general geometries will be considered. These are a mandrel that is cocked at the top of the motor case and a mandrel that is cocked at both the top and the bottom of the motor case. The following presents the details of the two geometries.

1. Mandrel Cocked With Respect to the Motor Case Top

In the case of the cocked mandrel the assumption is made that the axis of symmetry of the mandrel ( $\bar{Z}$-axis) is cocked with respect to the axis of symmetry of the motor case ( 2 -axis). In the case where the mandrel is cocked with the respect to the top of the motor case, the assumption is made that the coordinate systems of the motor case and the mandrel have the same origin. However, the $\bar{X}-\bar{Y}-\bar{Z}$ coordinate system is created by_rotating the $X-Y-Z$ system about the $Y$-axis. Therefore, the $X-, Z^{-}, \bar{X}-$, and $\bar{Z}$-axes are coplanar and the $Y-$ and $\bar{Y}$-axes are identical. The geometry is presented in Figure 5.


Figure 5. Cocked mandrel configuration (cocked at top).

In order to determine the geometry of a grain created with a cocked mandrel three simplifying assumptions are implied. These are:
a. Axial distances_along the propellant grain will be determined along the z -axis instead of the $\overline{\mathrm{z}}$-axis.
b. The propellant cross-section of the unburned portion in the $X-Y$ plane is circular instead of elliptical.
$\bar{X}-Y$ plane.
c. The propellant burns radially, in the $X-Y$ plane instead of the

These assumptions are justified by the fact that the angle between the $Z$ and $\bar{Z}$ axes (which is the same angle between the $X$ and $\bar{X}$ axes) will be very small and thus the cosine of the included angle will be very close to unity. In order for distances along the $\overline{\mathrm{z}}$-axis to exceed distances along the z -axis by more than $.1 \%$ the included angle must exceed $2.5^{\circ}$. This angle should be well within the region of mandrel misalignment that is normally encountered. Thus, because of the very small included angle the unburned propellant port should be essentially circular in the $X-Y$ plane. Also, this small included angle means that web distances burned along the $X$-axes are essentially unchanged when projected on the X-axis. And finally, the effects of assumptions a. and c. above tend to cancel each other and thus increase the accuracy.

The geometry of a propellant grain cast with a cocked mandrel can be considered to experience four distinct phases as the motor progresses from the initial state to motor burnout. These four phases are:

PHASE 1. The port of the propellant is totally circular. The short propellant web had not burned out at any axial cross-section.

PHASE 2. The short propellant web has burned out for crosssections in upper portion of the grain. The remainder of the grain has a circular port.

PHASE 3. The short propellant web had burned out for the entire length of the grain. There are no cross-sections for which total propellant burn out has occurred.

PHASE 4. The cross-section at the bottom of the motor has experienced total propellant burn out.

The next step is to consider the geometry of the motor during each of the following four phases:

PHASE 1

$$
0 \leq \tau \leq \tau_{1}
$$

Where

$$
\begin{align*}
& \tau_{I}=\frac{R_{c}-R(0)-\Delta X_{T}}{\left(1-\frac{\Delta X_{T} N_{t o p}}{L(0)}\right)}  \tag{22}\\
& \Delta X_{T}=\Delta x(Z=L(0)) \tag{23}
\end{align*}
$$

Where
$\tau_{1}$ is the web distance burned for short web burn out at the top of the grain.
$N_{\text {top }}=0$ If the top end is inhibited
$=1$ If the top end is uninhibited
and $\Delta \mathrm{X}_{\mathrm{T}}$ is the initial off-set of the mandrel axis at the top of the grain. The burning area of the motor is:

$$
\begin{equation*}
A_{b}(\tau)=P(\tau) L(\tau)+\left(N_{\text {bot }}+N_{t o p}\right) A_{c r}(\tau) \tag{24}
\end{equation*}
$$

Where

$$
\begin{equation*}
L(\tau)=L(0)-\tau\left(N_{\text {bot }}+N_{\text {top }}\right) \tag{25}
\end{equation*}
$$

$$
\begin{equation*}
N_{\text {bot }}=0 \text { If the bottom is inhibited } \tag{26}
\end{equation*}
$$

$=1$ If the bottom is uninhibited
and for all phases

$$
R(\tau) \text { is determined from Equation (1) }
$$

The free volume of the motor is given by Equation (21).
PHASE 2

$$
\tau_{1} \leq \tau<\tau_{2}
$$

Where

$$
\begin{equation*}
\tau_{2}=\frac{R_{c}-R(0)}{\left(1+\frac{N_{b o t} \Delta X_{T}}{L(0)}\right)} \tag{28}
\end{equation*}
$$

## Where

$\tau_{2}$ is the web distance burned for short web burn out at the bottom of the grain

The burning surface area of the motor is given by:

$$
\begin{gathered}
A_{b}(\tau)=P\left(\tau, z_{b o t}\right)\left(z_{u b}-z_{b o t}\right)+\int_{Z_{u b}}^{Z_{t o p}} P(\tau, z) d z \\
+N_{b o t} A_{c r}\left(\tau, z_{b o t}\right)+N_{\text {top }} A_{c r}\left(\tau, z_{\text {top }}\right)
\end{gathered}
$$

Where
$Z_{\text {bot }}$ - is the $Z$-coordinate of the bottom of the grain
$Z_{u b}$ - is the $Z$-coordinate at which the cross-section is at the exact point of short web burn out and
$Z_{\text {top }}-1 s$ the $Z$-coordinate of the top of the grain.
Where

$$
\begin{align*}
& \mathrm{Z}_{\text {bot }}(\tau)=\tau \mathrm{N}_{\text {bot }}  \tag{30}\\
& \mathrm{Z}_{\mathrm{ub}}(\tau)=\frac{\mathrm{L}(0)\left(\mathrm{R}_{\mathrm{c}}-\mathrm{R}(0)-\tau\right)}{\Delta \mathrm{X}_{\mathrm{T}}}  \tag{31}\\
& \mathrm{Z}_{\text {top }}(\tau)=\mathrm{L}(0)-\tau \mathrm{N}_{\text {top }}
\end{align*}
$$

and note that for all phases:

$$
\begin{equation*}
\Delta x(Z)=\Delta x_{T} \frac{z}{L(0)} \tag{33}
\end{equation*}
$$

The integral term can be approximated by applying the trapezoidal rule over 11 points:

$$
\begin{equation*}
\int_{Z_{u b}}^{Z_{\text {top }}} P(\tau, z) d z=\frac{\Delta z}{2} \quad \sum_{i=1}^{10}\left(P\left(\tau, z_{i}\right)+P\left(\tau, Z_{i}-\Delta Z\right)\right) \tag{34}
\end{equation*}
$$

Where:

$$
\begin{align*}
& \Delta z=\frac{z_{\text {top }}(\tau)-z_{u b}^{(\tau)}}{10}  \tag{35}\\
& z_{i}=z_{u b}+i(\Delta z) \quad i=1,2, \ldots, 10 \tag{36}
\end{align*}
$$

Thus, the burning surface area is given by:

$$
\begin{align*}
& A_{b}(\tau)=P\left(\tau, z_{b o t}\right)\left(z_{u b}-z_{b o t}\right)+\frac{\Delta z}{2} \sum_{i=1}^{10}\left(P\left(\tau, z_{i}\right)+P\left(\tau, z_{i}-\Delta z\right)\right) \\
& +N_{b o t} A_{c r}\left(\tau, Z_{b o t}\right)+N_{\text {top }} A_{c r}\left(\tau, z_{\text {top }}\right)
\end{align*}
$$

The free volume of the motor is given by:

$$
\begin{equation*}
V(\tau)=\pi R_{c}^{2} L(0)-A_{c r}\left(\tau, Z_{b o t}\right)\left(Z_{u b}-Z_{b o t}\right)-\int_{Z_{u b}}^{Z_{t o p}} A_{c r}(\tau, Z) d z \tag{38}
\end{equation*}
$$

This can be approximated by:

$$
\begin{align*}
& V(\tau)=\pi R_{c}^{2} L(0)-A_{c r}\left(\tau, Z_{b o t}\right)\left(Z_{u b}-Z_{b o t}\right) \\
& \left.-\frac{\Delta Z}{2} \sum_{i=1}^{10} A_{c r}\left(\tau, Z_{i}\right)+A_{c r}\left(\tau, Z_{i}-\Delta Z\right)\right) \tag{39}
\end{align*}
$$

The grain length is given in Equation (25).
PHASE 3

$$
\tau_{2} \leq \tau<\tau_{3}
$$

Where

$$
\begin{equation*}
\tau_{3}=\frac{R_{c}-R(0)}{\left(1-\frac{\Delta X_{T} N_{\text {top }}}{L(0)}\right)} \tag{40}
\end{equation*}
$$

Where
$\tau_{3}$ is the web distance burned for total propellant burn out at the bottom cross-section.

The burning surface area of the motor is given by:

$$
\begin{equation*}
A_{b}(\tau)=\int_{Z_{\text {bot }}}^{Z_{\text {top }}} P(\tau, z) d z+N_{\text {bot }} A_{c r}\left(\tau, Z_{\text {bot }}\right)+N_{\text {top }} A_{c r}\left(\tau, Z_{\text {top }}\right) \tag{41}
\end{equation*}
$$

This can be approximated by:

$$
\begin{align*}
& A_{b}(\tau)=\frac{\Delta Z}{2} \sum_{i=1}^{10}\left(P\left(\tau, z_{i}\right)+P\left(\tau, z_{i}-\Delta Z\right)\right)  \tag{42}\\
& \quad+N_{\text {bot }} A_{c r}\left(\tau, Z_{\text {bot }}\right)+N_{\text {top }} A_{c r}\left(\tau, z_{\text {top }}\right)
\end{align*}
$$

Where
$Z_{\text {bot }}(\tau)$ is determined from Equation (30) and
$z_{\text {top }}(\tau)$ is determined from Equation (32)

$$
\begin{equation*}
\Delta Z=\frac{\mathrm{z}_{\text {top }}(\tau)-\mathrm{z}_{\text {bot }}(\tau)}{10} \tag{43}
\end{equation*}
$$

and $z_{i}=Z_{\text {bot }}+i(\Delta Z) \quad i=1,2, \ldots ., 10$
The free volume of the motor is given by:

$$
\begin{equation*}
V(\tau)=\pi R_{c}^{2} L(0)-\int_{Z_{\text {bot }}}^{Z_{\text {top }}} A_{c r}(\tau, z) d z \tag{45}
\end{equation*}
$$

This can be approximated by:

$$
V(\tau)=\pi R_{c}{ }^{2} L(0)-\frac{\Delta Z}{2} \sum_{i=1}^{10}\left(A_{c r}\left(\tau, z_{i}\right)+A_{c r}\left(\tau, Z_{i}-\Delta z\right)\right)
$$

The grain length is given in Equation (25).

PHASE 4

$$
\tau_{3} \leq \tau<\tau_{\text {mbo }}
$$

where

$$
\begin{equation*}
\tau_{\text {mbo }}=\frac{\mathrm{R}_{\mathrm{c}}-\mathrm{R}(0)+\Delta \mathrm{X}_{\mathrm{T}}}{\left(1+\frac{\Delta \mathrm{x}_{\mathrm{T}} \mathrm{~N}_{\text {top }}}{\mathrm{L}(0)}\right)} \tag{47}
\end{equation*}
$$

Where
$\tau_{\text {mbo }}$ is the web distance burned for total motor propellant burn out.
The relationships for burning surface area and motor free volume are the same as those presented for Phase 3 with the exception that:

$$
\begin{equation*}
\mathrm{z}_{\text {bot }}(\tau)=\left(\tau-\mathrm{R}_{\mathrm{C}}-\mathrm{R}(0)\right) \frac{\mathrm{L}(0)}{\Delta X_{T}} \tag{48}
\end{equation*}
$$

The length of the propellant grain is given by:

$$
\begin{equation*}
\mathrm{L}(\tau)=\mathrm{z}_{\mathrm{top}}(\tau)-\mathrm{z}_{\mathrm{bot}}(\tau) \tag{49}
\end{equation*}
$$

2. Mandrel cocked with respect to both the motor case bottom and top

A variation of the cocked mandrel geometry can be achieved by considering the case where the mandrel is cocked at both the top and bottom of the motor case. In this case the $\overline{\mathrm{z}}$ - axis is created by rotating the z -axis about an axis which is parallel to the Y -axis and that passes through the centroid of the unburned propellant grain. This geometry is shown in Figure 6.

The geometry of propellant grain can be determined by applying the relationships derived from the situation where mandrel is cocked about the bottom of the motor case.

The burning surface area of the propellant grain is given by:

$$
\begin{equation*}
\mathrm{A}_{\mathrm{b}}(\tau)=2 \mathrm{~A}_{\mathrm{b}}\left(\tau, \mathrm{~L}(0), \mathrm{N}_{\mathrm{bot}}\right) \tag{50}
\end{equation*}
$$

Where:

$$
A_{b}\left(\tau, L(0), N_{b o t}\right)
$$

is the surface area determined for a propellant grain created by a mandrel cocked at the top only.


Figure 6. Cocked mandrel configuration (cocked at bottom and top).
The inputs to the burning surface area relationships are:

$$
\begin{equation*}
L(0)=\frac{L^{\prime}(0)}{2} \tag{51}
\end{equation*}
$$

and

$$
\begin{equation*}
N_{\text {bot }}=0 \tag{52}
\end{equation*}
$$

Where
$L^{\prime}(0)$ is the initial length of the propellant grain created by a mandrel cocked at both the bottom and top.

Likewise the free volume of motor is given by:

$$
\begin{equation*}
V(\tau)=2 V\left(\tau, L(0), N_{\text {bot }}\right) \tag{53}
\end{equation*}
$$

Note that these relationships apply only for the cases where either the top and bottom of the grain are both inhibited or both uninhibited.

Thus,

$$
\begin{equation*}
\text { if } N_{\text {top }}=0 \text { both ends are inhibited } \tag{54}
\end{equation*}
$$

$$
\begin{equation*}
\text { if } N_{\text {top }}=1 \text { both ends are uninhibited } \tag{55}
\end{equation*}
$$

Also, note that the geometry for grains which were generated by cocking the mandrel about horizontal axes located on various points on the $Z$-axis can also be determined from the previous relationships. These results can be obtained by adding the results for two appropriate motor geometries which were cocked at the top of the motor case.

## V. CONCLUSIONS

The mathematical model presented in this report provides a means to determine the geometrical profile of cylindrical port motors cast with misaligned mandrels. This model should serve as a valuable tool in determining the effect of mandrel misalignment on the pressure-time traces of ballistic test motors. In this application the model could be used to make some determination on the accuracy of burning rate data obtained from motors with various degrees of misalignments. Thus, this model could be used to establish a set of criteria for the accuracy of burning rate data obtained from cylindrical port ballistic test motors.

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## APPENDIX A

## HP-41C PROGRAM

The mathematical model presented in this report has been incorporated into a program for an HP-4IC calculator. This appendix is intended to provide all the information required to install and operate this program. This program, when installed on an HP-4l calculator system, will prove to be a useful analysis tool. The program as presented will provide the user with a convenient and accurate method for evaluating the geometry of misaligned cylindrical port motors. The following provides complete operating instructions, a set of sample problems, and a listing of the program. Also provided is all the required storage register and calculator status information needed to implement the program.

## A. Operating Instruction

In order to implement the program presented in this report the following equipment is required:

1 - HP-41CV calculator
or
1 - HP-41C calculator with 1 HP 82170A quad memory module
1 HP 83143A thermal printed/plotter
or
1-HP 83162A thermal printer/ploter with HP 82160A HP-IL module
To operate the program the printer should be mated with the calculator in the appropriate manner. The calculator should then be configured to size 43 and placed in the user mode. Table A-l provides a step by step key sequence required to operate this program.

TABLE A-1. Program Instructions

| STEP | INSTRUCTIONS | INPUT | FUNCTION | DISPLAY |
| :---: | :---: | :---: | :---: | :---: |
| 1. | Load Program. |  |  |  |
| 2. | Clear all resisters, |  | XEQ[ CLRG] |  |
| 3. | Initialize program. |  | $\Sigma+$ | THIS PROGRAM DETERMINES THE GEOMETRY OF CP GRAIN WITH AN OFF CENTER OR COCKED MANDREL |
| 4. | Key in case radius. | $\mathrm{R}_{\mathrm{c}}$ | R/S | COCKED? $\mathrm{Y}=1, \quad \mathrm{~N}=0$ |
| 5. | Indicate if the mandrel is cocked. | 1 or 0 | R/S |  |
| 5.a | If mandrel is cocked. | 1 | R/S | COCKED AT TOP ONLY $\mathrm{Y}=1, \mathrm{~N}=0$ |
| 5.b | If mandrel is not cocked go to Step 6. | 0 | R/S | LGRAIN = ? |
| 5.b.1 | If mandrel is cocked indicate if it is cocked at the top only. | 1 or 0 | R/S |  |
| 5.b.1.a | If mandrel is cocked at top only. | 1 | R/S | $\begin{aligned} & \text { BOTTOM BURNING } \\ & Y=1, \quad N=0 \end{aligned}$ |
| 5.b.1.b | If mandrel is not cocked at the top only, go to Step 5.b.2. | 0 | R/S | TOP BURNING $\mathrm{Y}=1, \quad \mathrm{~N}=0$ |
| 5.b. 2 | If mandrel is cocked at top only indicate if the bottom is burning. | $\mathrm{N}_{\text {bot }}$ | R/S | TOP BURNING $\mathrm{Y}=1, \quad \mathrm{~N}=0$ |
| 5.b.2.a | If mandrel is cocked indicate if the top is burning. | $\mathrm{N}_{\text {top }}$ | R/S | LGRAIN = ? |

Table A-1. Program Instructions - Continued

| STEP | INSTRUCTIONS | INPUT | FUNCTION | DISPLAY |
| :---: | :---: | :---: | :---: | :---: |
| 6. | Key in grain length. | L(0) | R/S | RGRAIN $=$ ? |
| 7. | Key in grain radius. | R(0) | R/S |  |
| $7 . a$ | If grain is cocked go to Step 8. |  |  | OFF SET $=$ ? |
| 7.b | If grain is not cocked. |  |  | NO. END BURN = ? |
| 7.b.1 | If grain is not cocked enter number of ends burning. | $\mathrm{N}_{\mathrm{eb}}$ | K/S | OFF SET = ? |
| 8. | Key in mandrel off set. | $\Delta \mathrm{X}$ or $\Delta \mathrm{X}_{T}$ | R/S | TAU START = ? |
| 9. | Key in starting web distance burned. | $\tau_{\text {start }}$ | R/S | TAU STOP = ? |
| 10. | Key in stopping web distance burned. | $\tau_{\text {stop }}$ | R/S |  |
| 10.a | If start and stop are equal go to step 11. |  |  |  |
| 10.b | If start and stop are not equal. |  |  | DELTA TAU \% = ? |
| 10.b. 1 | If start and stop are not equal enter the web distance increment then go to step 11 . | $\Delta \tau$ | R/S |  |
| 11. | Write program run information. |  |  |  |
| 11.a | If mandrel is not cocked. |  |  | GEOMETRY FOR <br> CP GRAIN WITH <br> AN OFFSET OF <br> X.XXXXX IN <br> AND X ENDS BURNING <br> SHORT WEB $=$ X. XXXXXX <br> MAX WEB $=\mathrm{X} . \mathrm{XXXXXX}$ |

Table A-1. Program Instructions - Continued

| STEP | INSTRUCTIONS INPUT | FUNCTION | DISPLAY |
| :---: | :---: | :---: | :---: |
| 11.b | If mandrel is cocked at the top and bottom. |  | GEOMETRY FOR COCKED <br> CP GRAIN WITH <br> AN OFF SET OF <br> X.XXXXX IN <br> AND X ENDS BURNING <br> SHORT WEB $=$ X. XXXXXX <br> MAX WEB $=$ X. XXXXXX |
| $11 . \mathrm{c}$ | If mandrel is cocked at the top only. |  | GEOMETRY FOR COCKED <br> AT TOP ONLY <br> CP GRAIN WITH <br> AN OFFSET OF <br> X.XXXXXX IN <br> AND X ENDS BURNING <br> SHORT WEB $=$ X. XXXXXX <br> MAX WEB $=$ X. XXXXXX |
| 12. | Display motor geometries for web distance burned values from $\tau_{\text {start }}$ to $\tau_{\text {stop }}$ in increments of $\Delta \tau$. Also display the geometry for the point of short web burn out. In addition program will stop at $\tau_{\text {mbo }}$ if $\tau_{\text {stop }}$ exceeds Tmbo. |  | ```TAU = X.XXXXXXX IN % Web = XX.XXXX% Ab = XXX.XXXX SQ In VOL = XXX.XXXX CU IN TAU = X.XXXXXX IN % Web = XX.XXXX% Ab = XXX. XXXX SQ IN VOL = XXX.XXXX CU IN SHORT WEB BURN OUT``` |
| 13. | (Optional) <br> Evaluate a single motor geometry. | $1 / \mathrm{X}$ | TAU $=$ ? |
| 14. | (Optional) Key in web distance burned to be evaluated, | R/S | ```TAU = X.XXXXXXX IN % WEB = XX.XXXX% Ab = XXX.XXXX SQ IN VOL = XXX.XXXX CU IN``` |
| 15. | (Optional) <br> Evaluate the same motor geometry with a new offset value, Return to step 8. | $\sqrt{x}$ | OFF SET = ? |
| 16. | To evaluate a new problem go to step 3. |  |  |

When operating the program, note that as long as the program registers are not cleared all input values are maintained until they are specifically replaced. If any portion of the input sequence is initiated, the previous value for any input variable will be retained if $R / S$ is entered after the respective prompt. Thus, for an input value to be changed at a prompt, a numeric entry must be made.

Another item that should be noted when operating the program is the value of $\Delta \tau$. If the mandrel is not cocked the value of $\Delta \tau$ is the input as a percentage of $\tau_{\text {pbo }}$. If the mandrel is cocked, $\Delta \tau$ is input as a percentage of $\tau_{m b o}$. In addition, if the mandrel is not cocked, the short web value that is output is $\tau_{\text {sw }}$ and the maximum web value is $\tau_{\text {pbo. }}$. If the mandrel is cocked the short web value is $\tau_{1}$ and the maximum web value is $\tau_{m b o}$.

## B. Sample Problems

With the operation of the program completely detailed, the next step is to demonstrate the use of the program on some sample problems. For a sample motor geometry the $2 \times 4$ ballistic test motor was chosen. The basic dimensions of this motor are as follows:

$$
\begin{aligned}
& \mathrm{R}_{\mathrm{C}}=1.00 \mathrm{in} . \\
& \mathrm{L}(0)=3.75 \mathrm{in} . \\
& \mathrm{R}(0)=.75 \mathrm{in} .
\end{aligned}
$$

In the first sample problem, the program exercised was for a grain configuration which was cast with a mandrel cocked at the top only. For this same geometry the "ONE" and "START" options were also demonstrated. The "START" program option allows the user to evaluate the same basic configuration with a different degree of mandrel misalignment. The "ONE" option allows the user to evaluate the present configuration at a single web distance burned. The program was also exercised for two other grain configurations, a grain cast with a mandrel cocked at both the top and bottom and a grain cast with a displaced mandrel. The complete details of these sample problems are as follows:

Mandrel Cocked at Top Only
XEQ -OFCNTR-

THIS PROCRAM DETERMINES THE GEOMETRY OF CP GRAIN WITH ON OFF CENTER OR COCKED MOWBREL

RCASE=?
1.000800909 RUN

COCKEB?
$Y=1, N=0$
1.000000006 RUN

COCKES AT TOP OHLY
$Y=1, N=8$
1.000900000 RUN

BOTTOM BURNING?
$Y=1, N=0$
1.090000008 RUN

TOP BURUING?
$Y=1, N=0$
1.800080008 run

LGRAIM=?
3.750000000 RUN

RGRAIN=?
.750000000 RUW
SFF SET=?
.940080088 RUN
TAU START=?
. 290008800 RUN
TAU STOP=?
.228080800 RUN
DELTA TRU $\%=$ ?
2.80800809 RUN

GEOMETRY FOR
COCKED
RT TOP OMLY
CP GRAIN HITH
AN OFFSET OF 0.84608 IN RMD 2. ENDS BURNING
SHORT UEB $=6.212264 \mathrm{IN}$
MAX MEB=0. 286939 IN

TAU $=0.28908 \mathrm{JW}$
\% HEB=69.7911 \%
$\mathrm{Ab}=20.6888 \mathrm{SO}$ IN
$v O L=10.7548 \mathrm{CU}$ IN

TRu=0. 285739 IN
\% WEB=71.7011 \%
$\mathrm{Ab}=29.5928 \mathrm{SQ}$ IN
YOL $=18.8731 \mathrm{CU} \mathrm{IN}$
$T A J=8.211478 \mathrm{IN}$
\% MEB=73.7811 \%
肘 $=29.5739$ SQ IN
$4 O L=18.9912 \mathrm{CU}$ IN
T $Q u=0.212264 \mathrm{iN}$
\% WEB=73.9753 \%
$A b=28.5713$ SQ IN
$V O L=11.0974 \mathrm{CU}$ IN
suort mea
BURN OUT
$T Q U=0.217216 \mathrm{IN}$
\% WE8=75.7811 \%
$\mathrm{R}_{\mathrm{B}}=20.2429 \mathrm{se}$ IN
VOL $=11.1886 \mathrm{CU}$ IN
TRU $=0.229990$ IN
\% HEB=76.6713 \%
$\mathrm{A}_{\mathrm{A}}^{\mathrm{b}=19.9221 \mathrm{SO} \text { IN } \mathrm{M}}$
$Y O L=11.1645 \mathrm{CU} \mathrm{IN}$
"START" for Mandrel Cocked at Top Only

KEQ -STRRT"
OFF SET=?
.835988888 RUN
TAU START=?
.230088800 RUN
TAU STOP=?
.249090809 RUN
IELTA TAU \% =?
5.080808006 RIUN

GEOMETRY FOR
COCKED
AT TOP ONLY
CP GRAIN WITH
AN OFFSET OF 8.83508 IN AND 2. ENDS BURNING SHORT HEB $=9.217826$ IN MAX MEB $=9.282365$ IN

TRU $=9.238080 \mathrm{IN}$ \% HEB=81.4556 \% Ab=18.7354 SQ IN $40 \mathrm{~L}=11.3627 \mathrm{CU}$ IN

TRU=9. 248989 IN \% MEB=84.9965 \% Ab=15.6999 SQ IN $Y 0 \mathrm{~L}=11.5358 \mathrm{CU}$ IN
"ONE" for Mandrel Cocked at Top Only

XEQ -OME"
TRU=?
.260009890 RUN

TRU=0. 260990 IN
\% MEB=92.0795 \% $\mathrm{Ab}=4.7974 \mathrm{SO} \mathrm{IN}$ $V O L=11.7486 \mathrm{CJ} \mathrm{IN}$

Mandrel Cocked at Top and Bottom

XEQ "OFCNTR"
THIS PROCRAM
DETERHINES THE GEOMETRY OF CP GRAIN HITH AN OFF CENTER OR COCKED MANDREL

RCASE =?
1.008008080 RUN

COCKED?
$Y=1, N=0$
1.000000080 RUN

COCKED AT TOP ONLY
$Y=1, N=8$
0.008068088 RUN

TOP BURNING?
$Y=1, N=8$
1.898908808 RUN

LGRHIN $=$ ?
3.750809808 RUN

RCRAIN $=$ ?
.758800800 RUN

OFF SET=?
.840068008 RUN
TAU START=?
. 280808080 RUN
TAU STOP=?
.220890000 RUN
DELTA TAU $\%=$ ?
2.880800800 RUN

GEOMETRY FOR COCKED
CP GRAIN HITH
AN OFFSET OF 0.04800 IN
AND 2. ENDS BURNING
SHORT $\mathrm{MEB}=8.214578 \mathrm{IN}$
HAX $\mathrm{HEB}=8.283943 \mathrm{IN}$

TRU $=8.200608 \mathrm{IN}$ \% MEB=78.4368 \% $\mathrm{Pb}=20.6888 \mathrm{SO} \mathrm{IM}$ $40 \mathrm{~L}=10.7548 \mathrm{CU}$ IN

TRU $=8.285679$ IN \% HEB=72.4368 \% $\mathrm{Rb}=26.5922 \mathrm{SB} \mathrm{IN}$ $Y O L=18.8718 \mathrm{CU}$ IN

TRU $=0.211358 \mathrm{IN}$
\% MEB=74.4368 \%
$\mathrm{Ab}=29.5743 \mathrm{SO} \mathrm{IN}$
YOL $=10.9887 \mathrm{CU}$ It'
TROU=0.214578 IN
\% WEB=75.5788 \%
Ab=28.5636 SQ IN
YOL $=11.8558 \mathrm{CU}$ IN
SHORT HEB
BURN OUT

TRU=0.217937 IN
\% MEB=76.4368 \%
$\mathrm{Ab}=28.4475 \mathrm{SQ}$ IN
YOL $=11.1054 \mathrm{CU}$ IN
TAU $=9.228080$ IN
\% HEB=77.4885 \%
Ab=20.1830 SQ IN
YOL=11.1656 CU IN

THIS PROGRAM DETERMINES THE GEOMETRY OF CP GRAIN HITH AN OFF CENTER OR COCKED mandrel

## RCASE=?

1.909808880 RUN

COCKED?
$Y=1, N=0$
6.868888006 RUN

LGRAIN=?
3.756800080 RUN

RGRAIN=?
.750800089 RUN
NO. END BURN=?
2.098900909 RUN

OFF SET=?
.848808008 RUN
TRU START=?
.288098988 RUN
TAU STOP=?
.220808006 RUN
DELTA TAU $\%=$ ?
2.008880908 RIN

GEOMETRY FOR
CP GRAIN HITH
AH OFFSET OF 8.04809 IN
and 2. ENDS BURNING
SHORT UEB=0.218808 IN
MAX MEB $=8.298008$ IN

TRU=8.208006 IN \% MEB=68.9655 \% $A b=28.6888 \mathrm{SQ}$ IN $Y O L=10.7548$ CU IN

TAU=0.285806 IN
\% ME8=70.9655 \%
Ab=28.5918 SQ IN
$Y O L=10.8743 \mathrm{CU}$ IN
TRU=0. 210095 IN
\% MEB=72.4138 \% $\mathrm{Ab}=28.5787 \mathrm{SO}$ IN $40 L=10.9688 \mathrm{CU}$ IN

SHORT HEB BURN OUT

TRU=0.211680 IN
\% MEB=72.9655 \%
$\mathrm{Ab}=18.7221$ SQ IN
YOL $=10.9917$ CU IN
TRU $=0.217400 \mathrm{IN}$
\% MEB=74.9655 \% Ab $=16.5252 \mathrm{SE} \mathrm{IN}$ $Y O L=11.0932 \mathrm{CU}$ IN

TAU=0.228068 IN \% HEB=75.8621 \% $A b=15.8357 \mathrm{SQ}$ IN YOL=11. 1353 CU IN

C．Installation Information
With the operational aspects of the program presented，the next step is to provide the information required to install the program on an $\mathrm{HP}-41 \mathrm{C}$ calculator system．Presented below is a complete listing of the program．From this listing the program can be directly keyed into the calculator．To facilitate an understanding of the program listing，the storage resister assignments are pre－ sented in Table A－2，and to aid in the installation and operation of the program information about the required calculator status is presented in Table A－3．

```
    01*LBL "OFC
NTR*
    OZ FIX9
    03 CF 22
    04 CF 01
    05 CF 02
    06 CF 03
    07 ADV
    08 "THIS PF
OGRAM "
    09 AVIEW
    10 "DETERMI
NES THE*
    11 AYIEW
    12 "GEOMETR
Y OF"
    13 AVIEW
    14 "CP GRAI
H NITH"
    15 AVIEM
    16."PN OFF
CENTER"
    17 AVIEW
    18 "OR COCK
ED"
    19 AVIEW
    20 "MANDREL Input
*
    21 AVIEW
    22 ADV
    23 - RCASE=?
*
    24 PROMPT
    25 FS? 2Z
    26 STO 09
    27 CF 22
    28 COCKED?
29 AVIEM
30 . Y=1,N=
```

日＂
31 PROMPT
32 FS？22
$335 T 040$
34 CF 22
351
36 RCL 40
$37-$
$38 \quad x<=6 ?$
39 SF 02
40 FC？ 92
41 GTD 76
42 －COCKED
日T TOF O＂
43 ＂トHL＇r＂
44 AVIEW
$45 \cdot \gamma=1, N=$
$0 \cdot$
46 PROMPT
47 FS？ 22
48 STO 42
49 CF 22
501
51 RCL 42
52 －
$53 \quad x<=0 ?$
54 SF 0．3
550
56 FC？ 03
$575 T 037$
58 FC？ 03
59 GT0 65
60 －BOTTOM
BURHIHG？＂
61 AVIEW
$62 \cdot Y=1, N=$
0
63 PROMPT
64 FS？22

$115 x<=0 ?$
116 GTO 11
1176
118 STO 17
119 \& LBL 11
$120+L B L$-STA
RT"
121 CF 1
122 - DFF SET
=?
123 PROMPT
124 FS? 22
125 STO 38
126 CF 22
127 RCL 38
$1285 T 008$
129 "TRU STA
$R T=$ ? ${ }^{-\quad}$
130 PROMPT
131 FS? 22
$1325 T 021$
133 CF 22
134 . TAU STD
P=?
Input
135 PROMPT
136 FS? 22
137 STO 22
138 CF 22
1390
$1405 T 000$
141 XEQ "GEO
142 FS? 02
143 XEQ "GEO
$2 "$
144 RCL 22
145 RCL 21
146 -
$147 x<=0 ?$
148 GTO 20
149 - DELTA T
คU \%=?
150 PROMPT
151 FS? 22
$1525 T 020$
153 FC? 22
154 GTO 20
155 RCL 20
156100
157
158 RCL 16
159 *
160 FC? 02
161 GTO 89
162 RCL 16

Evaluate the same motor configuration with a new mandrel offset

| 163 | / |  | EB: $=$ |  |
| :---: | :---: | :---: | :---: | :---: |
| 164 | RCL 27 |  | 212 | ARCL $X$ |
| 165 | * |  | 213 | $\cdots \vdash$ IH" |
| 166 | LBL 89 |  | 214 | AVIEW |
| 167 | STO 20 |  | 215 | RCL 16 |
| 168 | LBL 20 |  | 216 | FS? 日2 |
| 169 | CF 22 |  | 217 | RCL 27 |
| 170 | ADV |  | 218 | - MAX WEE |
| 171 | *GEOMETR |  | = ${ }^{\text {- }}$ |  |
| $Y \mathrm{Y}$ | R ${ }^{\text {" }}$ |  | 219 | AREL $X$ |
| 172 | AVIEN |  | 220 | *ト IN" |
| 173 | FC? 02 |  | 221 | AVIEW |
| 174 | GTO 79 |  | 222* | LBL 95 |
| 175 | "COCKED |  | 223 | FIX 9 |
| 176 | AVIEW |  | 224 | ADV |
| 177 | FC? 03 |  | 225 | RCL 21 |
| 178 | GT0 79 |  | 226 | STO 06 |
| 179 | * AT TOP |  | 227 | FC? 92 |
| ONLY |  |  | 228 | RCL 10 |
| 180 | AVIEW | Output | 229 | FS? 02 |
| 181 | LBL 79 | information | 230 | RCL 24 |
| 182 | -CP GRAI | on the case | 231 | - |
| N WI | ITH* | being evaluated | 232 | CHS |
| 183 | AVIEW |  | 233 | ¢< $=0$ ? |
| 184 | FIX 5 |  | 234 | SF 01 |
| 185 | RCL 08 |  | 235 | LEL 30 |
| 186 | FS? 02 |  | 236 | FS? 02 |
| 187 | RCL 38 |  | 237 | XEQ "GEO |
| 188 | - AN OFFS |  | 2 |  |
| ET | OF - |  | 238 | FS? 02 |
| 189 | ARCL $X$ |  | 239 | GT0 81 |
| 190 | $\cdots$ - IH. |  | 240 | KEQ "GEO |
| 191 | AVIEW |  | - |  |
| 192 | FIX 0 |  | 241 | - 81 |
| 193 | RCL 17 |  | 242 | XEQ "OUT |
| 194 | FC? 02 |  | PUT |  |
| 195 | GTO 80 |  | 243 | RCL 22 |
| 196 | 2 |  | 244 | RCL 21 |
| 197 | RCL 36 |  | 245 | - |
| 198 | RCL 37 |  | 246 | $x<=0 \%$ |
| 199 | + |  | 247 | STOP |
| 200 | FC? 03 |  | 248 | RCL 20 |
| 201 | * |  | 249 | $5 T+60$ |
| 202 | - 186 |  | 250 | RCL 22 |
| 203 | "PND - |  | 251 | RCL A0 |
| 204 | ARCL X |  | 252 | - |
| 205 | $\cdots$ - ENDS |  | 253 | $x<=6 ?$ |
| BURN | NING* |  | 254 | GT0 35 |
| 206 | PVIEW |  | 255 | RCL 16 |
| 207 | FIX 6 |  | 256 | FS? 62 |
| 208 | RCL 19 |  | 257 | RCL 27 |
| 209 | FS? 02 |  | 258 | RCL OQ |
| 210 | RCL 24 |  | 259 | - |
| 211 | - SHORT W |  | 260 | $x<=0 ?$ |


| 261 | GTO 31 |
| :---: | :---: |
| 262 | FS? 01 |
| 263 | GTO 30 |
| 264 | RCL 10 |
| 265 | FS? 02 |
| 266 | RCL 24 |
| 267 | RCL 88 |
| 268 | - |
| 269 | X< $=0$ |
| 276 | GTO 32 |
| 271 | GTO 30 |
| 272 | LBL 31 |
| 273 | RCL 16 |
| 274 | FS? 02 |
| 275 | RCL 27 |
| 276 | STO 08 |
| 277 | FS? 82 |
| 278 | XEQ -GEO |
| 2" |  |
| 279 | FS? 02 |
| 289 | GTO 82 |
| 281 | XEQ -GEO |
| 282 | LBL 82 |
| 283 | XEQ |
| PUT ${ }^{\text { }}$ |  |
| 284 | GTO 75 |
| 285 | LBL 32 |
| 286 | RCL 00 |
| 287 | STO 23 |
| 288 | SF 11 |
| 289 | RCL 10 |
| 290 | FS? 02 |
| 291 | RCL 24 |
| 292 | RCL 00 |
| 293 |  |
| 294 | $\mathrm{X}=6$ ? |
| 295 | GTO 47 |
| 296 | RCL 10 |
| 297 | FS? 02 |
| 298 | RCL 24 |
| 299 | STO 0 |
| 390 | FS? 02 |
| 301 | XEQ -GEO |
| $2 \times$ |  |
| 302 | FS? 02 |
| 303 | GTO 83 |
| 304 | XEQ -GEO |
| 305 | LBL 83 |
| 306 | XEQ -OUT |
| PUT |  |
| 307 | -LBL 47 |
| 398 | RCL 23 |
| 309 | STO 00 |
| -:0 | QDV |



| 360 |  |  | 411 | $x<0$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 360 | AVIEW |  | 412 | GTO | 00 |
| 361 | RCL 14 |  | 413 | RCL | 09 |
| 362 | "VOL= ${ }^{\text {P }}$ |  | 414 | 欠イ2 |  |
| 363 | ARCL $X$ |  | 415 | RCL | 03 |
| 364 | "F CU IM |  | 416 | ¢T2 |  |
| 365 |  |  | 417 | - |  |
| 365 | PVIEM |  | 418 | P I |  |
| 366 | FIX 9 |  | 419 | * |  |
| 367 | RTN | - - - - - - - | 420 | RCL | 17 |
| 368 * | LBL "GED |  | 421 | * |  |
| 369 |  |  | 422 | RCL | 03 |
| 369 | RCL O1 |  | 423 | 2 |  |
| 370 | RCL DE |  | 424 | * |  |
| 371 | + |  | 425 | PI |  |
| 372 | ST0 03 |  | 426 | * |  |
| 373 | RCL 02 |  | 427 | RCL | 04 |
| 374 | RCL BE |  | 428 |  |  |
| 375 | RCL 1 ? |  | 429 | + |  |
| 376 | * |  | 430 | STO | 13 |
| 377 | FC? 02 |  | 431 | RCL | 09 |
| 378 | FC? 02 |  | 432 | $x+2$ |  |
| 379 | STO 04 |  | 433 | PI |  |
| 380 | RCL 09 |  | 434 | * |  |
| 381 | RCL 08 |  | 435 | RCL | 02 |
| 382 | + |  | 436 | * |  |
| 383 | RCL O1 |  | 437 | RCL | 09 |
| 384 | STO 16 |  | 438 | K+2 |  |
| 385 | $5 T 016$ | Calculate the | 439 | RCL | 03 |
| 386 | RCL 16 | geometry for | 440 | 8T2 |  |
| 387 | FS? 02 | a motor cast | 441 | - |  |
| 388 | RCL 27 | with an offset | 442 | PI |  |
| 389 | RCL OO | mandrel | 443 | * |  |
| 390 | - |  | 444 | RCL | 04 |
| 391 | CHS |  | 445 | * | 04 |
| 392 | x $=0 ?$ |  | 446 | - |  |
| 393 | GTO 01 |  | 447 | 570 | 14 |
| 394 | LBL 75 |  | 448 | 360 |  |
| 395 | ADV |  | 449 | STO | 06 |
| 396 | - MOTOR B |  | 450 | STO | 07 |
| URNE | D ${ }^{\text {P }}$ |  | 451 | RCL | 03 |
| 397 | PVIEW |  | 452 | 2 |  |
| 398 | "OUT" |  | 453 | * |  |
| 399 | AVIEW |  | 454 | PI |  |
| 400 | ADV |  | 455 | * |  |
| 401 | STOP |  | 456 | $5 T 0$ | 05 |
| 402 | LBL 01 |  | 457 | RCL | 09 |
| 403 | RCL 09 |  | 458 | X+2 |  |
| 404 | RCL 01 |  | 459 | RCL | 03 |
| 405 | PCL 08 |  | 460 | x+2 |  |
| 406 | RCL 08 |  | 461 | - |  |
| 407 | -TO 10 |  | 462 | PI |  |
| 488 | STO 10 |  | 463 | * |  |
| 489 | RCL OO |  | 464 | STO | 19 |
| 410 | - |  | 465 | RTH |  |


| 466* | LBL | 00 | $5 こ 1$ | < |
| :---: | :---: | :---: | :---: | :---: |
| 467 | RCL | 03 | 522 | ATAN |
| 468 | X+2 |  | 523 | 2 |
| 469 | RCL | 09 | 524 | * |
| 470 | x+2 |  | 525 | STO 07 |
| 471 | - |  | 526 | GTO 94 |
| 472 | RCL | 08 | 527 | LBL 93 |
| 473 | XT2 |  | 528 | RCL 18 |
| 474 | - |  | 529 | RCL 15 |
| 475 | 2 |  | 530 | / |
| 476 | - |  | 531 | CHS |
| 477 | RCL | 98 | 532 | ATAN |
| 478 | / |  | 533 | 2 |
| 479 | STO | 15 | 534 | * |
| 480 | XT2 |  | 535 | CHS |
| 481 | CHS |  | 536 | 360 |
| 482 | RCL | 09 | 537 | + |
| 483 | Xt2 |  | 538 | STO 07 |
| 484 | + |  | 539 | LBL 04 |
| 485 | PBS |  | 540 | RCL 15 |
| 486 | SQRT |  | 541 | RCL 88 |
| 487 | STO | 18 | 542 | + |
| 488 | RCL | 88 | 543 | $x \neq 0$ ? |
| 489 | RCL | 03 | 544 | GTO 05 |
| 490 | + |  | 545 | 180 |
| 491 | RCL | 09 | 546 | STO 06 |
| 492 | + |  | 547 | GTO 97 |
| 493 | 2 |  | 548 | LBL 05 |
| 494 | / |  | 549 | RCL 15 |
| 495 | STO | 11 | 558 | RCL 88 |
| 496 | RCL | 03 | 551 | + |
| 497 | - |  | 552 | $x<0$ ? |
| 498 | RCL | 11 | 553 | GTO 06 |
| 499 | RCL | 08 | 554 | RCL 08 |
| 500 | - |  | 555 | RCL 15 |
| 591 | * |  | 556 | + |
| 502 | RCL | 11 | 557 | 1/x |
| 593 | RCL | 09 | 558 | RCL 18 |
| 504 | - |  | 559 | * |
| 505 | * |  | 560 | ATAN |
| 506 | RCL | 11 | 561 | 2 |
| 507 | * |  | 562 | * |
| 508 | SART |  | 563 | STO 06 |
| 509 | STO | 12 | 564 | GTO 07 |
| 510 | RCL | 15 | 565 | -LBL 06 |
| 511 | $x \neq 0$ ? |  | 566 | RCL 08 |
| 512 | GTO | 02 | 567 | RCL 15 |
| 513 | 189 |  | 568 | + |
| 514 | STO | 07 | 569 | CHS |
| 515 | GTO | 04 | 570 | 1 <x |
| 516 | LBL | 02 | 571 | RCL 18 |
| 517 | x<0? |  | 572 | * |
| 518 | GTO | 03 | 573 | PTAN |
| 519 | RCL | 18 | 574 | 2 |
| 520 | RCL | 15 | 575 | * |


| 576 | CHS |  | 031 | ＊ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 577 | 360 |  | 632 | CHS |  |
| 578 | $+$ |  | 633 | PI |  |
| 579 | STO | 06 | 634 | RCL 09 |  |
| 588 | LBL | 07 | 635 | XT2 |  |
| 581 | PI |  | 636 | ＊ |  |
| 582 | 180 |  | 637 | RCL 02 |  |
| 583 | ， |  | 638 | ＊ |  |
| 584 | RCL | 06 | 639 | $+$ |  |
| 585 | ＊ |  | 640 | STO 14 |  |
| 586 | RCL | 03 | 641 | RCL 19 |  |
| 587 | ＊ |  | 642 | x＋2 |  |
| 588 | STO | 05 | 643 | RCL 07 |  |
| 589 | RCL | 09 | 644 | ＊ |  |
| 596 | X＋2 |  | 645 | RCL 03 |  |
| 591 | RCL | 07 | 646 | X＋2 |  |
| 592 | ＊ |  | 647 | RCL 06 |  |
| 593 | RCL | 03 | 648 | ＊ |  |
| 594 | メイ2 |  | 649 | － |  |
| 595 | RCL | 86 | 650 | PI |  |
| 596 | ＊ |  | 651 | ＊ |  |
| 597 | － |  | 652 | 360 |  |
| 598 | PI |  | 653 | － |  |
| 599 | ＊ |  | 654 | RCL 12 |  |
| 680 | 360 |  | 655 | 2 |  |
| 601 | － |  | 656 | ＊ |  |
| 602 | RCL | 12 | 657 | ＋ |  |
| 603 | 2 |  | 658 | STO 19 |  |
| 694 | ＊ |  | 659 | RTH |  |
| 605 | ＋ |  | 660 | －LBL－ONE |  |
| 606 | RCL | 17 | ．． |  | Calculate the |
| 607 | ＊ |  | 661 | CF 11 | motor geometry |
| 698 | RCL | 04 | 662 | －TAU＝？${ }^{\text {－}}$ | for a single |
| 689 | RCL | 05 | 663 | PROMPT | web distance |
| 618 | ＊ |  | 664 | FS？ 22 | burned |
| 611 | ＋ |  | 665 | STO 21 |  |
| 612 | STO | 13 | 666 | CF 22 |  |
| 613 | RCL | 09 | 667 | RCL 21 |  |
| 614 | ×＋2 |  | 668 | STO 22 |  |
| 615 | RCL | 07 | 669 | 100 |  |
| 616 | ＊ |  | 670 | STO 20 |  |
| 617 | RCL | 83 | 671 | GTO 95 |  |
| 618 | XT2 |  | 672 | LBL－GEO |  |
| 619 | RCL | 06 | $2 \times$ |  |  |
| 629 | ＊ |  | 673 | RCL 89 |  |
| 621 | － |  | 674 | RCL 01 |  |
| 622 | PI |  | 675 | － |  |
| 623 | ＊ |  | 676 | RCL 38 |  |
| 624 | 360 |  | 677 |  |  |
| 625 | ＇ |  | 678 | 1 |  |
| 626 | RCL | 12 | 679 | RCL |  |
| 627 | 2 |  | 689 | RCL 36 |  |
| 628 | ＊ |  | 681 | ＊ |  |
| 629 | ＋ |  | 6日 | FíL 日 |  |
| 639 | RCL | $\underline{4}$ | $6 \%$ |  |  |


| 684 | - |  | Calculate the | 739 | RCL | 37 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 685 | / |  | geometry for | 740 |  |  |
| 686 | STO | 24 | a motor cast | 741 | RCL | 00 |
| 687 | RCL | 37 | with a cocked | 742 | * |  |
| 688 | RCL | 38 | mandrel. | 743 | - |  |
| 689 | * |  |  | 744 | STO | 04 |
| 690 | RCL | 02 |  | 745 | RCL | 36 |
| 691 | , |  |  | 746 | RCL | 37 |
| 692 | 1 |  |  | 747 | + |  |
| 693 | + |  |  | 748 | P I |  |
| 694 | 1/X |  |  | 749 | * |  |
| 695 | RCL | 09 |  | 759 | RCL | 09 |
| 696 | RCL | 01 |  | 751 | $x+2$ |  |
| 697 | - |  |  | 752 | RCL | 03 |
| 698 | * |  |  | 753 | x+2 |  |
| 699 | STO | 25 |  | 754 | - |  |
| 700 | RCL | 37 |  | 755 | * |  |
| 701 | RCL | 38 |  | 756 | 2 |  |
| 702 | * |  |  | 757 | PI |  |
| 703 | RCL | 02 |  | 758 | * |  |
| 704 | < |  |  | 759 | RCL | 03 |
| 705 | CHS |  |  | 760 | * |  |
| 706 | 1 |  |  | 761 | RCL | 04 |
| 707 | + |  |  | 762 | * |  |
| 708 | $1 / X$ |  |  | 763 | + |  |
| 709 | RCL | 09 |  | 764 | STO | 13 |
| 718 | RCL | 01 |  | 765 | RCL | 09 |
| 711 | - |  |  | 766 | X+2 |  |
| 712 | * |  |  | 767 | RCL | 93 |
| 713 | STO | 26 |  | 768 | $x+2$ |  |
| 714 | RCL | 09 |  | 769 | - |  |
| 715 | RCL | 01 |  | 770 | P I |  |
| 716 | R |  |  | 771 | * |  |
| 717 | RCL | 38 |  | 772 | RCL | 04 |
| 718 | + |  |  | 773 | * |  |
| 719 | RCL | 38 |  | 774 | CHS |  |
| 720 | RCL | 36 |  | 775 | RCL | 89 |
| 721 | * |  |  | 776 | X+2 |  |
| 722 | RCL | 02 |  | 777 | PI |  |
| 723 | / |  |  | 778 | * |  |
| 724 | 1 |  |  | 779 | RCL | 02 |
| 725 | + |  |  | 780 | * |  |
| 726 | / |  |  | 781 | + |  |
| 727 | STO | 27 |  | 782 | STO | 14 |
| 728 | RCL | 08 |  | 783 | RTN |  |
| 729 | RCL | 24 |  | 784 | LBL | 71 |
| 730 | - |  |  | 785 | RCL | 06 |
| 731 | $x>0 ?$ |  |  | 786 | RCL | 25 |
| 732 | GTO | 71 |  | 787 | - |  |
| 733 | RCL | 01 |  | 788 | $x>0 ?$ |  |
| 734 | RCL | 08 |  | 789 | GTO | 72 |
| 735 | + |  |  | 790 | RCL | 00 |
| 736 | STO | 03 |  | 791 | RCL | 37 |
| 737 | RCL | 02 |  | 792 | * |  |
| 738 | RCL | $3 E$ |  | 793 | STO | 29 |


| $\begin{aligned} & 794 \\ & 795 \end{aligned}$ | $\begin{aligned} & \text { RCI } \\ & \text { RC: } \end{aligned}$ | $09$ $01$ |
| :---: | :---: | :---: |
| 796 | R |  |
| 797 | RCL | 00 |
| 798 | - |  |
| 799 | RCL | 38 |
| 850 | / |  |
| 801 | RCL | 02 |
| 802 | * |  |
| 803 | ST0 | 29 |
| 804 | STO | 41 |
| 805 | RCL | 02 |
| 806 | RCL | 日0 |
| 807 | RCL | 36 |
| 898 | * |  |
| 809 | - |  |
| 810 | STO | 30 |
| 811 | RCL | 28 |
| 812 | - |  |
| 813 | ST0 | 04 |
| 814 | XEQ | - ARE |
| A" |  |  |
| 815 | RCL | 29 |
| 816 | RCL | 28 |
| 817 | - |  |
| 818 | STO | 23 |
| 819 | RCL | 31 |
| 820 | * |  |
| 821 | ST+ | 13 |
| 822 | RCL | 23 |
| 823 | RCL | 34 |
| 824 | * |  |
| 825 | ST- | 14 |
| 826 | RTN |  |
| 827 | LBL | 72 |
| 828 | RCL | E8 |
| 829 | RCL | 26 |
| 830 | - |  |
| 831 | $x>0 ?$ |  |
| 832 | GTO | 73 |
| 833 | RCL | 37 |
| 834 | RCL | 00 |
| 835 | * |  |
| 836 | ST0 | 28 |
| 837 | STO | 41 |
| 838 | RCL | 62 |
| 839 | RCL | 00 |
| 840 | RCL | 36 |
| 841 | * |  |
| 842 | - |  |
| 843 | STO | 38 |
| 544 | RCL | 28 |
| 345 | - |  |
| 946 | STO | 24 |
| : $: 47$ | XEG | - APE |

A"
848 RTM
849*LBL 73
856 RCL 80
851 RCL 27
852 -
$853 x>0 ?$
854 GTD 75
855 RCL 80
856 RCL 09
857 -
858 RCL 01
859 +
860 RCL 02
861 *
862 RCL 38
863 /
864 5T0 28
865 STO 41
866 RCL 02
867 RCL 10
868 RCL 36
869 *
$870-$
871 ST0 30
872 RCL 28
873 -
874 STO 04
875 XEQ *ARE
${ }^{\boldsymbol{A}}$
876 RTN
87ア・LBL - RRE
A-
878 RCL 28
879 RCL 02
880
881 RCL 38
882 *
883 5T0 08
884 XEQ -GEO
-
885 RCL 05
886 STO 31
887 RCL 19
888 STO 34
889 RCL 3R
898 RCL 02
891
892 RCL 38 Integrate to
893 *
894 ST3 08
895 XEG -GED
$\cdot$
BGEFEL BS Approximation
obtain surface area and volume using a trapezoidal rule. pproximation

| 898 | RCL 19 | 935 | RCL 19 |
| :---: | :---: | :---: | :---: |
| 899 | STO 35 | 936 | ST+ 39 |
| 909 | RCL 31 | 937 | RCL 17 |
| 901 | RCL 33 | 938 | ST+ 41 |
| 902 | + | 939 | CF 21 |
| 903 | 2 | 946 | YIEW 41 |
| 904 | < | 941 | SF 21 |
| 905 | STO 32 | 942 | RCL 41 |
| 906 | RCL 34 | 943 | 1.00001 |
| 907 | RCL 35 | 944 | * |
| 908 | + | 945 | RCL 30 |
| 909 | 2 | 946 | - |
| 910 | / | 947 | $x<0 ?$ |
| 911 | STO 39 | 948 | GTO 99 |
| 912 | CF 21 | 949 | RCL 17 |
| 913 | VIEW 41 | 950 | ST* 32 |
| 914 | SF 21 | 951 | ST* 39 |
| 915 | RCL 30 | 952 | RCL 34 |
| 916 | RCL 41 | 953 | RCL 37 |
| 917 | - | 954 | * |
| 918 | 10 | 955 | ST+ 32 |
| 919 | - | 956 | RCL 35 |
| 929 | STO 17 | 957 | RCL 36 |
| 921 | ST+ 41 | 958 | * |
| 922 | CF 21 | 959 | ST+ 32 |
| 923 | VIEW 41 | 969 | RCL 32 |
| 924 | SF 21 | 961 | STO 13 |
| 925 | LBL 99 | 962 | RCL 09 |
| 926 | RCL 41 | 963 | メナ2 |
| 927 | RCL 02 | 964 | PI |
| 928 | ' | 965 | * |
| 929 | RCL 38 | 966 | RCL 02 |
| 936 | * | 967 | * |
| 931 | STO 08 | 968 | RCL 39 |
| 932 | XEQ -GEO | 969 | - |
| " |  | 970 | STO 14 |
| 933 | RCL 05 | 971 | RTN |
| 934 | Sit 32 | 972 | END |

TABLE A-2. Register Assignments

| RESISTER | VARIABLE | UNITS |
| :---: | :---: | :---: |
| 00 | $\tau$ | in |
| 01 | R(0) | in |
| 02 | L(0) | in |
| 03 | R | in |
| 04 | L | in |
| 05 | P | in |
| 06 | $\theta_{1}$ | deg |
| 07 | $\theta_{2}$ | deg |
| 08 | $\Delta \mathrm{X}$ | in |
| 09 | $\mathrm{R}_{\mathrm{c}}$ | in |
| 10 | $\tau_{\text {sw }}$ | in |
| 11 | S | $\mathrm{in}_{2}$ |
| 12 | $A_{1}$ | $\mathrm{in}_{2}$ |
| 13 | $A_{b}$ | $\mathrm{in}_{3}$ |
| 14 | V | in ${ }^{3}$ |
| 15 | $\mathrm{X}_{\mathrm{I}}$ | in |
| 16 | $\tau_{\text {pbo }}$ | in |
| 17 | $\mathrm{N}_{\text {eb }}, \Delta Z$ | NA, in |
| 18 | $\mathrm{Y}_{\mathrm{I}}$ | $\mathrm{in}_{2}$ |
| 19 | $\mathrm{A}_{\mathrm{cr}}$ | $1 n^{2}$ |
| 20 | $\Delta \tau$ | in |
| 21 | ${ }^{\text {c start }}$ | in |
| 22 | $\tau_{\text {stop }}$. | in |
| 23 | used | NA |
| 24 | $\tau_{1}$ | in |
| 25 | $\tau_{2}$ | in |
| 26 | $\tau_{3}$ | in |
| 27 | $\tau_{\text {mbo }}$ | in |
| 28 29 | $\mathrm{Z}_{\text {ub }}$ | in |
| 30 | $z_{\text {top }}$ | in |
| 31 | $\mathrm{P}\left(\mathrm{Z}_{\text {bot }}\right)$ | in |
| 32 | $\mathrm{P}\left(\mathrm{Z}_{\mathbf{u b}}\right)$, P | in, in |
| 33 | $\mathrm{P}\left(\mathrm{Z}_{\text {top }}\right)$ | in 2 |
| 34 35 | $A_{c r}\left(Z_{\text {bot }}\right)$ | in ${ }^{2}$ |
| 36 | $\underset{\mathrm{N}_{\text {top }}}{\mathrm{A}_{\mathrm{Cr}}}\left(\mathrm{Z}_{\text {toD }}\right)$ | NA |
| 37 | N bot | NA |
| 38 | $\Delta \mathrm{X}_{\mathrm{T}}$ | $\mathrm{In}_{2}$ |
| 39 | $\sum \mathrm{A}_{\mathrm{c}} \mathrm{r}$ | $1 n^{2}$ |
| 40 | used | $\mathrm{NA}_{2}$ |
| 41 | 2 | in ${ }^{2}$ |
| 42 | used | NA |

TABLE A-3. Calculator Status

| Calculator mode |  | USER |  |
| :---: | :---: | :---: | :---: |
| Size |  | 43 |  |
| Program registers |  | 276 |  |
| Total registers |  | 319 |  |
|  | I+ | OFCNTR |  |
|  | 1/X | ONE |  |
|  | X | START |  |
| $\begin{aligned} & \infty \\ & \vec{Z} \\ & \tilde{0} \\ & \text { in } \\ & \text { N } \\ & \text { N } \end{aligned}$ | Flag No. | Set Flag Indicates | Cleared Flag Indicates |
|  | 01 | Web distance burned has exceeded the short web | Web distance burned has not exceeded the short web |
|  | 02 | Mandrel is Cocked | Mandrel is not cocked |
|  | 03 | Mandrel is cocked at top only | Mandrel is cocked top and bottom |

## MISALIGNED $2 \times 4$ MOTOR

The $2 \times 4$ ballistic test motor is the basic burning rate characterization motor employed by the Propulsion Directorate. This motor has a cylindrical port and in normal applications has both end surfaces uninhibited.

Initial port radius: $R(0)=.75$ in

Initial grain length: $\mathrm{L}(0)=3.75 \mathrm{in}$

Case Radius:

$$
\mathrm{R}_{\mathrm{c}}=1.00 \mathrm{in}
$$

This motor was used as an example to demonstrate the application of the misaligned motor geometry model. For this motor the burning surface area histories were generated for geometries reflecting a perfectly aligned mandrel, a displaced mandrel, a mandrel cocked at the top, and a mandrel cocked at both the top and bottom. For the three modes of mandrel misalignment, surface area histories were generated for $\Delta \mathrm{X}_{\mathrm{T}}$ values of 0.00 in, , 0.01 in, , $0.02 \mathrm{in} ., 0.03$ in., 0.04 in., 0.05 in., 0.06 in., 0.07 in., 0.08 in., 0.09 in., and 0.10 in. All these surface area histories were generated using the HP-41C calculator and the previously detailed program. The burning surface area history for an aligned $2 \times 4$ motor grain is presented in Figure B-1. The burning surface area histories for a $2 \times 4$ motor cast with a displaced mandrel is presented in figure B-2. The burning surface area histories for a $2 \times 4$ motor cast with a mandrel cocked at the top is presented in Figure B-3. The burning surface area histories for a $2 \times 4$ motor cast with a mandrel cocked at both the top and bottom is presented in Figure $B-4$.


Figure B-1. Burning surface area history of $2 \times 4$ motor cast with no mandrel misalignment.


Figure B-2. Burning surface area history of $2 \times 4$ motor cast with a displaced mandrel.


Figure B-3. Burning surface area history of $2 \times 4$ motor cast with a mandrel cocked at the top.


$$
\bar{\square}
$$

Figure B-4. Burning surface area history of $2 \times 4$ motor cast with a mandrel cocked at the top and bottom.
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