NAVWEPS REPORT 7827
NOTS TP 2838
COPY 313

HANDBOOK OF EQUATIONS FOR MASS AND AREA PROPERTIES OF VARIOUS GEOMETRICAL SHAPES

Compiled by

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ABSTRACT. This publication is a compilation of equations for moments of inertia, centroidal distances, radii of gyration, and other mathematical properties related to solids, thin shells, thin rods, plane areas, and ogival shapes.

U.S. NAVAL ORDNANCE TEST STATION

China Lake, California

April 1962

U. S. NAVAL ORDNANCE TEST STATION

AN ACTIVITY OF THE BUREAU OF NAVAL WEAPONS

C. BLENMAN, JR., CAPT., USN
Commander

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Technical Director

FOREWORD

A need has existed for a comprehensive handbook containing properties of various geometrical shapes to be used by design engineers at governmental agencies.

It is the purpose of this publication to supply technical personnel with information concerning these mathematical properties in a complete volume that includes moments of inertia, centroidal distances, volumes, areas, and radii of gyration of solids, thin shells, thin rods, plane areas, and ogival shapes. In addition, examples of various types are included.

The work of compiling, organizing, and preparing this publication was done at the U. S. Naval Ordnance Test Station in September 1961 under Bureau of Naval Weapons Task Assignment RM3773-009/216-1/F008-22-002 of 22 September 1961.

This handbook was reviewed for technical accuracy by Genge Industries, Inc., of Ridgecrest, California. Suggested additions or criticism will be appreciated. The information contained herein is to be released at the working level only.

Released by
G. F. CLEARY, Head,
Air-to-Surface Weapons Div.
12 December 1961

Under authority of
F. H. KNEMEYER, Head,
Weapons Development Dept.

ACKNOWLEDGMENT

The author is indebted to J. W. Odle and to the following publishers for permission to use material in compiling this handbook: American Institute of Steel Construction; Harvard University Press; Schaum Publishing Co.; Society of Aeronautical Weight Engineers, Inc.; The Industrial Press; and Wiley and Sons, Inc. A complete bibliographical list of sources is given at the back of this publication.

NOTS Technical Publication 2838
NAVWEPS Report 7827

Published by . . . . Publishing Division
Technical Information Department
Collation . . . Cover, 48 leaves, abstract cards
First printing (April 1962) . 245 numbered copies
Second printing (June 1963) . 100 numbered copies
Security classification . . . . . . UNCLASSIFIED

U. S. NAVAL ORDNANCE TEST STATION
CHINA LAKE, CALIFORNIA

IN REPLY REFER TO:
7511/FD:tah
21 September 1966

From: Commander, U. S. Naval Ordnance Test Station
To: Distribution List of NAVWEPS Report 7827, NOTS TP 2838

Subj: NAVWEPS Report 7827 (NOTS TP 2838), Handbook of Equations for Mass and Area Properties of Various Geometrical Shapes, dated April 1962; transmittal of errata sheets for

Encl: (1) Errata sheets (sheets 1-4) dated September 1966 for subject report

1. It is requested that the corrections and comments presented in the enclosed errata sheets be incorporated in NAVWEPS Report 7827, NOTS TP 2838. The enclosed material supersedes the previously distributed errata sheet dated 12 May 1965.

C. E. Van Hagan
C. E. VAN HAGAN
By direction

ERRATA

Page 81: Under the heading "Moment of Inertia About the Base Plane," the equations are correct for moment of inertia about the base plane; however, to obtain moment of inertia about a base diameter axis, add "$+\frac{1}{2} I_A$" to the right-hand side of each of the three equations for I_B.

Page 82: Under the heading "Moment of Inertia About the Base Plane," the equations are correct for moment of inertia about the base plane; however, to obtain moment of inertia about a base diameter axis, add "$+\frac{1}{2} I_A$" to the right-hand side of each of the two equations for I_B.

Page 88: In the underscored heading, change "the Base Plane" to read "a Base Diameter Axis."

In the equations below the figure, change "I_B" to "I_{BA}," three places.

Page 89: In the figure, change the dimension "L" to "h" and "b" to "D."

In the last underscored heading, change "the Base Plane" to read "a Base Diameter Axis."

In the equations at the top and at the bottom of the page, change "I_B" to "I_{BA}."

Page 90: In line 2, change "$a = L/R = \sin \phi$" to read "$a = h/R = \sin \phi$."

In the third equation below the figure, change "I_P" to read "I_A."

In the fourth equation below the figure, change "I'_{TB}" to read "I_B." The fourth equation changed as above to read "$I_B = \ldots$" is correct for the moment of inertia about the base plane; however, to obtain moment of inertia about a base diameter axis, add "$+\frac{1}{2} I_A$" to the right-hand side of the equation.

COMMENTS

1. Inertia equations give answers in inches to the fifth power.

2. Do not use a slide rule to calculate ogival properties. At least six significant figures must be calculated for each term within the brackets given with the ogive equations. Therefore, it is advised to use a desk calculator or other type of computer to establish the desired accuracy.

3. Central axis: The central axis is the symmetrical center line axis of the ogive sometimes referred to as the polar, or polar longitudinal axis.

4. Base diameter axis: The base diameter axis denotes an orthogonal transverse axis which intersects the central axis at the base of the ogive. This is commonly referred to as the transverse axis.

5. Base plane: The base plane denotes a plane passing through the base of the shape and normal to its center line axis.

6. Moment of inertia about the base plane: The moment of inertia about the base plane can be computed by subtracting one-half the value of the moment of inertia about the central axis from the value of the moment of inertia about a base diameter axis. Conversely, the moment of inertia about a base diameter axis can be computed by adding one-half the value of the moment of inertia about the central axis to the value of the moment of inertia about the base plane. Mathematically,

$$I_B = I_{BA} - \frac{1}{2} I_A$$

and

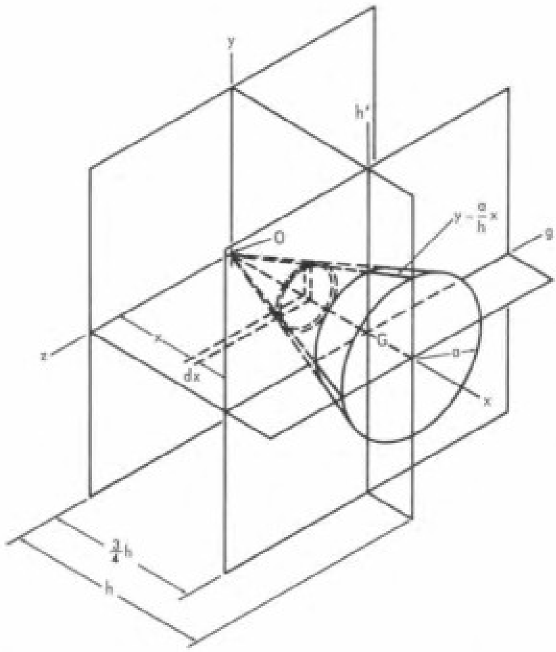
$$I_{BA} = I_B + \frac{1}{2} I_A$$

where

I_B = moment of inertia about the base plane
I_{BA} = moment of inertia about a base diameter axis
I_A = moment of inertia about the central axis

7. Example (from Calculus, by Edward S. Smith, Meyer Salkover, and Howard K. Justice, New York, John Wiley and Sons, Inc., 1947, Article 113, Example 5, pp. 317-318; used by permission of the publisher): The following example is given to show the methods for obtaining moments of inertia about planes and axes of a solid of revolution.

Example: Find the moment of inertia of the volume of a right circular cone of altitude h and base-radius a with respect to the following planes and axes parallel to the base: (i) a plane through the apex; (ii) an axis through the apex; (iii) an axis through the centroid; (iv) a plane through the centroid.



(i) Choosing three mutually perpendicular coordinate planes as shown in the figure, we proceed to find I_{yz} by integration. Using discs as elements of volume we have

$$I_{yz} = \int_0^h \pi y^2 dx \cdot x^2 = \pi \frac{a^2}{h^2} \int_0^h x^4 dx$$

$$= \frac{\pi a^2 h^3}{5}$$

(ii) By symmetry, the moment of inertia of the volume of the cone with respect to any axis through the apex and parallel to the base is equal to I_z, which may be expressed in the form

$$I_z = I_{xz} + I_{yz}$$

where I_{yz} is given and I_{xz} remains to be found.

Evidently $I_{xz} = I_{xy}$, and hence

$$I_{xz} = \frac{1}{2}(I_{xy} + I_{xz})$$
$$= \frac{1}{2} I_x$$
$$= \frac{\pi a^4 h}{20}$$

Substituting the values of I_{xz} and I_{yz}, we obtain

$$I_z = \frac{\pi a^2 h}{20}(a^2 + 4h^2)$$

(iii) The distance from the apex to the centroid of the cone is $\frac{3}{4}h$. Hence, if V represents the volume of the cone and a g-axis is drawn through the centroid G parallel to the z-axis, we have

$$I_g = I_z - V(\tfrac{3}{4}h)^2$$

Therefore

$$I_g = \frac{\pi a^2 h}{80}(4a^2 + h^2)$$

Obviously this result is the moment of inertia of the volume of the cone with respect to any axis drawn parallel to the base through the centroid.

(iv) With respect to the gh′- plane, drawn through the centroid G and parallel to the base, the moment of inertia of the volume of the cone is

$$I_{gh'} = I_{yz} - V(\tfrac{3}{4}h)^2$$
$$= \frac{\pi}{80} a^2 h^3$$

CONTENTS

LIST OF DIAGRAMS

The following is a list of the geometrical shapes for which diagrams and equations are given.

NOMENCLATURE

An effort has been made to typify symbols for thickness, height, and base width for all shapes and plane areas considered. However, in some instances, additional nomenclature has been introduced for descriptive purposes. It is advisable to refer to the diagram associated with each item.

A Area, in^2

I Moment of inertia. The unit is generally expressed as follows: for a solid, $lb\text{-}ft^2$, $slug\text{-}ft^2$, etc; for a plane area, in^4, ft^4, etc.

I_p Polar moment of inertia

I_x Moment of inertia about the x-axis

I_y Moment of inertia about the y-axis

I_z Moment of inertia about the z-axis

I_{xy} Product of inertia in the x-y plane

I_{xz} Product of inertia in the x-z plane

I_{yz} Product of inertia in the y-z plane

k_x Radius of gyration about the x-axis

k_y Radius of gyration about the y-axis

k_z Radius of gyration about the z-axis

L Length, in. Where t (thickness) or d (diameter) of a thin rod is constant, V, W, and m are proportional. Statical moments and moments of inertia of the area or body may be functions of L

m Mass, lb or slugs

V Volume, in^3

W Weight, lb

$\bar{x}$ Centroidal distance along the x-axis, in.

$\bar{y}$ Centroidal distance along the y-axis, in.

$\bar{z}$ Centroidal distance along the z-axis, in.

ρ Mass density factor, lb/in^3; m/V

INTRODUCTION

This handbook is published for the convenience of those whose work requires the use of equations of mass and area properties for various geometrical shapes. It is hoped that this compilation will be an aid to technical personnel and will eliminate the need for searching through many handbooks and tables for a particular mathematical property.

In certain sections of this handbook, equations for moments of inertia and centroidal distances are developed through the use of the calculus, which, it is believed, will serve as a supplementary method for finding information not included herein. Other, simplified, forms for calculating properties are included tbat can be used in lieu of integral calculus.

As an aid in finding a particular item, the diagrams and accompanying equations for the various shapes are grouped under the following headings shown at the tops of the respective pages: solids, thin shells, thin rods, plane areas, and ogival shapes.

Certain structural shapes such as channels, I-beams, angles, and T-sections are omitted because these are normally found in a construction manual such as the AISC steel construction manual. However, variations of these sections, without fillets and bulbs, are included.

A separate section is devoted to the properties of ogival shapes, which are commonly used in the design of missile nose cones. The solid and thin-shelled tangent ogives are included because of their frequent use.

Special Notes

1. A thin-shelled body is one in which $t < (L/30)$, where t is the gage thickness of the material and L is the length, or radius, perpendicular to the axis of rotation measured at the maximum diameter.

2. A thin rod is one in which $L \geqq 30d$, where L is the length of the rod and d is the diameter of the rod.

3. Elliptic-area formulas may be used for circular complements such as half circles and quarter circles by substituting $a = b = R$.

4. Weight moments of inertia for plane areas can be obtained by multiplying the area moment of inertia by the area mass, M, and then dividing by the section area.

5. Linear dimensions are in inches in the sections that follow.

6. In most cases involving integration, cartesian, or rectangular, coordinates are used. Should the need arise to use polar coordinates for ease of integration of special integrals, it is advised that a review of a calculus text or similar reference be made. Also, double integration methods, if used, will in many instances reduce calculation time.

SOLIDS

SUMMARY OF EQUATIONS FOR MASS AND VOLUME PROPERTIES

Centroid by Integration (Homogeneous Mass)

$$\bar{x} = \frac{\int x\,dV}{\int dV}, \qquad \bar{y} = \frac{\int y\,dV}{\int dV}, \qquad \bar{z} = \frac{\int z\,dV}{\int dV}$$

Center of Gravity of a Coplanar System of Particles

$$\bar{x} = \frac{My}{m} = \frac{\Sigma Mx}{M}, \qquad \bar{y} = \frac{Mx}{m} = \frac{\Sigma My}{M}$$

Center of Gravity of a Non-coplanar System of Particles

$$\bar{x} = \frac{Myz}{m} = \frac{\Sigma Mx}{M}, \qquad \bar{y} = \frac{Mxz}{m} = \frac{\Sigma My}{M}, \qquad \bar{z} = \frac{Mxy}{m} = \frac{\Sigma Mz}{M}$$

Moment of Inertia

$$I = \int r^2\,dm$$

Product of Inertia

$$I_{xy} = \int xy\,dm, \qquad I_{yz} = \int yz\,dm, \qquad I_{xz} = \int xz\,dm$$

Transfer of Axes

$$I = \bar{I} + md^2$$

$$k^2 = \bar{k}^2 + d^2$$

Radius of Gyration

$$k = \sqrt{I/m}$$

Centroid of a Composite Solid Body

$$\bar{x} = \frac{\Sigma W\bar{x}}{W} = \frac{\Sigma L\bar{x}}{L} = \frac{\Sigma V\bar{x}}{V}$$

$$\bar{y} = \frac{\Sigma W\bar{y}}{W} = \frac{\Sigma L\bar{y}}{L} = \frac{\Sigma V\bar{y}}{V}$$

$$\bar{z} = \frac{\Sigma W\bar{z}}{W} = \frac{\Sigma L\bar{z}}{L} = \frac{\Sigma V\bar{z}}{V}$$

CENTER OF GRAVITY OF A SYSTEM OF PARTICLES

Two systems of particles exist that will be defined as coplanar and non-coplanar.

Coplanar Particles

The first system, coplanar particles, can be resolved into a common mass located at such a position that the moment of its mass with respect to the x-axis would be equal to the moment-sum about the x-axis, and that the moment of its mass with respect to the y-axis would be equal to the moment-sum about the y-axis. The two coordinates presented would then locate the point that represents the center of gravity of the system in the x-y plane.

Therefore,

$$\bar{x}m = M_y \qquad \text{and} \qquad \bar{y}m = M_x$$

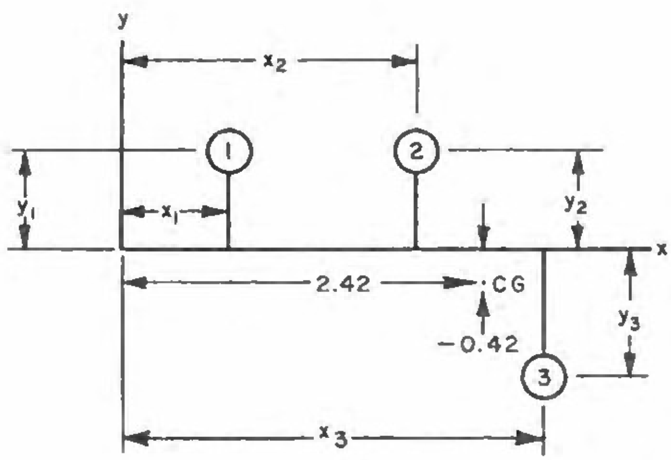
or

$$\bar{x} = \frac{M_y}{m} = \frac{m_1 x_1 + m_2 x_2 + \cdot \cdot \cdot + m_n x_n}{m_1 + m_2 + \cdot \cdot \cdot + m_n} = \frac{\Sigma mx}{\Sigma m}$$

and

$$\bar{y} = \frac{M_x}{m} = \frac{m_1 y_1 + m_2 y_2 + \cdot \cdot \cdot + m_n y_n}{m_1 + m_2 + \cdot \cdot \cdot + m_n} = \frac{\Sigma my}{\Sigma m}$$

Example.



$x_1 = 1$ in., $\quad y_1 = 1$ in., $\quad m_1 = 1$ lb

$x_2 = 2$ in., $\quad y_2 = 2$ in., $\quad m_2 = 2$ lb

$x_3 = 3$ in., $\quad y_3 = -2$ in., $\quad m_3 = 4$ lb

$$\bar{x} = \frac{(1)(1) + (2)(2) + (4)(3)}{1 + 2 + 4} = 2.428 \text{ in.}$$

$$\bar{y} = \frac{(1)(1) + (2)(2) + (4)(-2)}{1 + 2 + 4} = -0.429 \text{ in.}$$

Note. Care must be taken to account for the proper signs when calculating the moment-sums.

Non-coplanar Particles

The second system, non-coplanar particles, can be resolved into a common mass located at a point in space represented by the coordinates $\bar{x}$, $\bar{y}$, and $\bar{z}$, which are readily calculated by the procedure used in the coplanar system and with the inclusion of the third reference-plane coordinate.

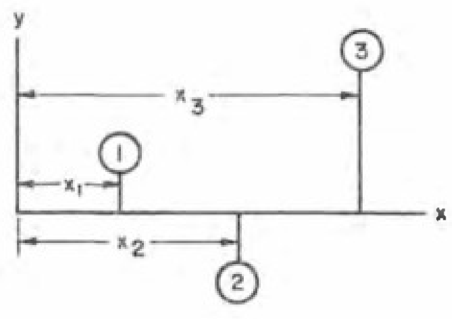
Thus,

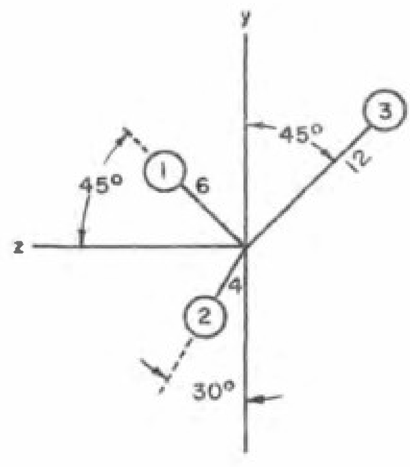
$$\bar{x} = \frac{Myz}{M} = \frac{m_1 x_1 + m_2 x_2 + \cdot \cdot \cdot + m_n x_n}{m_1 + m_2 + \cdot \cdot \cdot + m_n} = \frac{\Sigma mx}{\Sigma m}$$

$$\bar{y} = \frac{Mxz}{M} = \frac{m_1 y_1 + m_2 y_2 + \cdot \cdot \cdot + m_n y_n}{m_1 + m_2 + \cdot \cdot \cdot + m_n} = \frac{\Sigma my}{\Sigma m}$$

$$\bar{z} = \frac{Mxy}{M} = \frac{m_1 z_1 + m_2 z_2 + \cdot \cdot \cdot + m_n z_n}{m_1 + m_2 + \cdot \cdot \cdot + m_n} = \frac{\Sigma mz}{\Sigma m}$$

Example.





$x_1 = 2$ in., $y_1 = 6 \cos 45° = 4.24$, $z_1 = 6 \cos 45° = 4.24$

$x_2 = 4$ in., $y_2 = -4 \cos 30° = -3.46$, $z_2 = 4 \cos 60° = 2$

$x_3 = 6$ in., $y_3 = 12 \cos 45° = 8.48$, $z_3 = -12 \cos 45° = -8.48$

$m_1 = 1$ lb, $m_2 = 2$ lb, $m_3 = 3$ lb

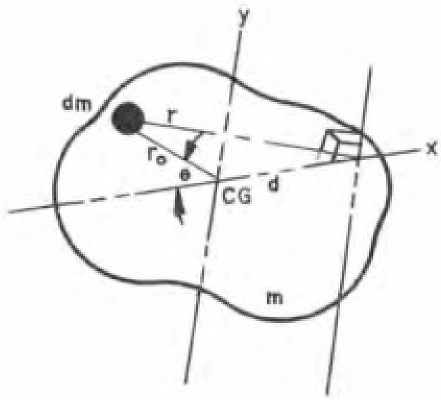
$$\bar{x} = \frac{(1)(2) + (2)(4) + (3)(6)}{6} = 4.66 \text{ in. (ans.)}$$

$$\bar{y} = \frac{(1)(4.24) + (2)(-3.46) + (3)(8.48)}{6} = 3.79 \text{ in. (ans.)}$$

$$\bar{z} = \frac{(1)(4.24) + (2)(2) + (3)(-8.48)}{6} = -2.87 \text{ in. (ans.)}$$

The resulting coordinates of 4.66, 3.79, and -2.87 inches fix the location of the system's center of gravity.

TRANSFER OF AXES ON A SOLID BODY



Let the radial distances from the two axes to any element of mass dm equal r and r_0, with the separation of the axes being d; applying the law of cosines, $r^2 = r_0^2 + d^2 + 2r_0 d \cos \theta$. The definition of the mass moment of inertia gives

$$I = \int r^2 dm = \int (r_0^2 + d^2 + 2r_0 d \cos \theta) dm$$

$$= \int r_0^2 dm + d^2 \int dm + 2d \int r_0 \cos \theta dm$$

Since the y-coordinate of the center of gravity with respect to an origin at θ is zero, the third integral drops out, leaving

$$I = \bar{I} + md^2$$

RADIUS OF GYRATION

The radius of gyration, k, of a body with respect to any axis is defined as the distance from the axis at which the mass may be conceived to be concentrated and to have the same moment of inertia with respect to the axis as does the actual whole, or distributed, mass.

Mathematically, $k = \sqrt{I/m}$ by definition. Substitution into $I = \bar{I} + md^2$ results in $k^2 = \bar{k}^2 + d^2$, which provides a method for transferring the centroidal radius of gyration to a parallel axis on the same body.

PRODUCT OF INERTIA

Generally, a three-dimensional body has three moments of inertia about the three mutually perpendicular coordinate axes and three products of inertia about the three coordinate planes.

The product of inertia of the body with respect to a pair of coordinate planes is the algebraic sum of the products obtained by multiplying the mass of each element of the body by its coordinates with reference to these planes. The value of the product of inertia can be positive, negative, or zero.

Mathematically, the products of inertia about the three planes are expressed as

$$I_{xy} = \int xy\,dm, \qquad I_{yz} = \int yz\,dm, \qquad I_{xz} = \int xz\,dm$$

where dm is an element of mass. Or, the product of inertia may be calculated for an area where the two rectangular coordinate axes provide tbe system on which the computation is based, in the form

$$I_{xy} = \int xy\,dA$$

where dA is an element of area, and x and y are the respective distances from the axes to the elements of area.

An application of the product of inertia may be seen on page 53 covering the properties of an angle.

CENTROIDS OF COMPOSITE VOLUMES

The determination of the centroid of a composite solid shape can be calculated by the application of the moment principle, in which the basic relationship takes the form

$$(w_1 + w_2 + w_3 + \cdot\cdot\cdot)\bar{X} = w_1\bar{x}_1 + w_2\bar{x}_2 + w_3\bar{x}_3 + \cdot\cdot\cdot$$

where w represents the weight of each part, $\bar{X}$ represents the x-coordinate of the center of gravity of the total body, and $\bar{x}$ represents the center of gravity of the individual parts. The resulting basic relationships are, therefore,

$$\bar{X} = \frac{\Sigma w\bar{x}}{W}, \qquad \bar{Y} = \frac{\Sigma w\bar{y}}{W}, \qquad \bar{Z} = \frac{\Sigma w\bar{z}}{W}$$

In each of the above relationships, weight values may be replaced with values for length, area, or volume, depending on the shape of the object.

It is likely that interest will be found in the determination of the centroidal distance of a hollow geometrical shape such as the frustum of a cone; the applicable equations will then be

$$\bar{X} = \frac{\Sigma v\bar{x}}{V}, \qquad \bar{Y} = \frac{\Sigma v\bar{y}}{V}, \qquad \bar{Z} = \frac{\Sigma v\bar{z}}{V}$$

where it is obvious that only one of these relationships is valid for a symmetrical body.

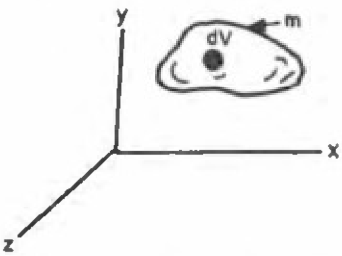
For hollow objects, or objects with holes in them, it should be realized that the void volume must be subtracted, as in the case of a bollow cone frustum.

$$\bar{X} = \frac{\Sigma v\bar{x}}{V} = \frac{(V\bar{x})_T - (V\bar{x})_H}{V_T - V_H}$$

where $(V\bar{x})_T$ represents the total solid frustum and $(V\bar{x})_H$ represents the inner, or hollow, frustum.

CENTROID OF A VOLUME

By expansion of the eoneepts used in the resolution of a system of partieles, it is apparent that a summation, by integration, of a differential element of a body, leads to the determination of the centroid of the body.



If the body is homogeneous, the density of the body, ρ, will be considered eonstant. Therefore, the element of mass is

$$dm = \rho dV$$

and, for the entire body,

$$m = \int \rho dV$$

Using previous equations and substituting,

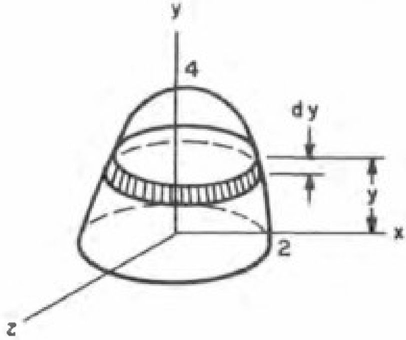
$$\bar{x} = \frac{\int x dm}{m} = \frac{\int \rho x dV}{\int \rho dV} = \frac{\int x dV}{\int dV}$$

$$\bar{y} = \frac{\int y dm}{m} = \frac{\int \rho y dV}{\int \rho dV} = \frac{\int y dV}{\int dV}$$

$$\bar{z} = \frac{\int z dm}{m} = \frac{\int \rho z dV}{\int \rho dV} = \frac{\int z dV}{\int dV}$$

Therefore, it ean be seen that the first moments of each summation are $V\bar{x}$, $V\bar{y}$, and $V\bar{z}$ for a homogeneous body.

Example. The eentroid of the solid generated by revolving the area of the half parabola $y = 4 - x^2$ about the y-axis may be determined as follows.



$$dV = \pi x^2 dy$$

$$\int dV = \int_0^4 \pi(4 - y)dy$$

$$\int dV = \pi\int_0^4 (4 - y)dy$$

$$V = \pi\left[4y - (y^2/2)\right]_0^4 = \pi(16 - 8) = 8\pi$$

$$V\bar{y} = \int y dV$$

$$= \int_0^4 \pi y x^2 dy = \pi\int_0^4 (4y - y^2)dy = \pi\left[2y^2 - (y^3/3)\right]_0^4$$

$$= \pi\left[32 - (64/3)\right] = 32\pi/3$$

Therefore,

$$\bar{y} = \frac{32\pi/3}{8\pi} = \frac{32}{24} = \frac{4}{3} \text{ (ans.)}$$

and $\bar{z} = \bar{x} = 0$, by symmetry.

MASS MOMENT OF INERTIA

The inertial resistance to rotational acceleration is that property of a body which is commonly known as its mass moment of inertia.

If a body of mass m is allowed to rotate about an axis at an angular acceleration α, an element of this mass, dm, will have a component of acceleration tangent to the circular path of $r\alpha$, with the tangential force on the element being $r\alpha dm$. Since the distance to the element is r, the resulting moment on the force equals $r^2\alpha dm$.

Integrating the elements of the body,

$$I = \int r^2 dm$$

an expression is obtained that is known as the mass moment of inertia of the body, where α is dropped out because it is constant for a given rigid body.

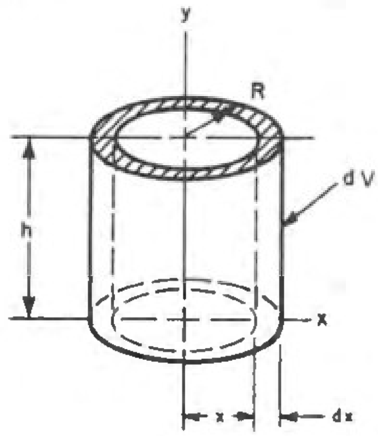
If the body is of constant mass density, the differential, dm, may be replaced with ρdV, since $dm = \rho dV$, and the following expression results

$$I = \rho\int r^2 dV$$

The units of mass moment of inertia are commonly expressed as $lb\text{-}ft\text{-}sec^2$ or $slug\text{-}ft^2$, or, dimensionally, ML^2.

Examples.

RIGHT CIRCULAR CYLINDER



The mass moment of inertia for a right circular cylinder is found by the use of $I = \int r^2 dm$ and the equation for elemental cylinders, as follows

$$dm = \rho dV = \rho 2\pi x h dx$$

$$I_y = \rho \int_0^R (x^2)(2\pi x) h dx$$

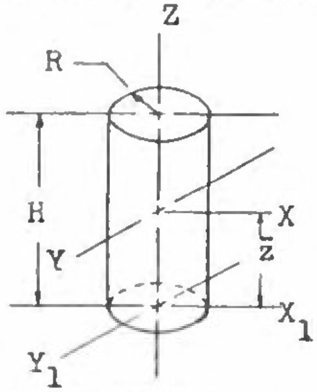
$$= 2\pi\rho h \int_0^R x^3 dx$$

$$= \frac{2\pi\rho h}{4}\left[x^4\right]_0^R = \frac{\rho\pi h R^4}{2}$$

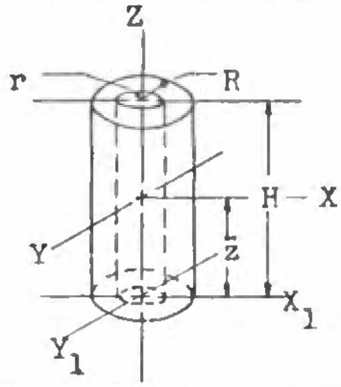
$$\rho = \frac{M}{V} = \frac{M}{\pi R^2 h}$$

$$I_y = \frac{M}{\pi R^2 h}\left(\frac{\pi h R^4}{2}\right) = \frac{MR^2}{2} \text{ (ans.)}$$

RIGHT CIRCULAR CYLINDER



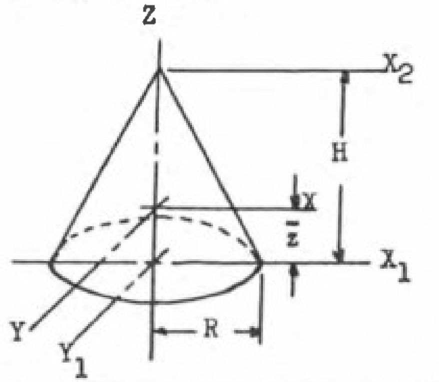
HOLLOW RIGHT CIRCULAR CYLINDER



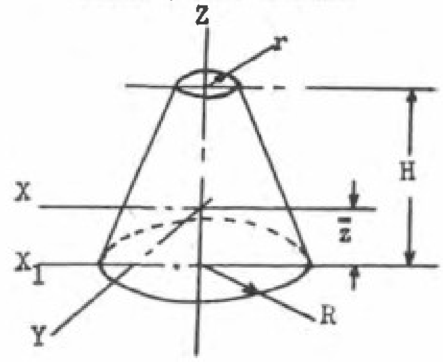
	RIGHT CIRCULAR CYLINDER	HOLLOW RIGHT CIRCULAR CYLINDER
VOLUME	$\pi R^2 H$	$\pi H(R^2 - r^2)$
CENTROID	$\bar{z} = \frac{H}{2}$	$\bar{z} = \frac{H}{2}$
WEIGHT MOMENT OF INERTIA	$I_x = I_y = \frac{W}{12}(3R^2 + H^2)$ $I_{x_1} = I_{y_1} = \frac{W}{12}(3R^2 + 4H^2)$ $I_z = \frac{WR^2}{2}$	$I_x = I_y = \frac{W}{12}\left[3(R^2 + r^2) + H^2\right]$ $I_{x_1} = I_{y_1} = W\left[\frac{R^2 + r^2}{4} + \frac{H^2}{3}\right]$ $I_z = \frac{W}{2}(R^2 + r^2)$
RADIUS OF GYRATION	$k_x = k_y = 0.289\sqrt{3R^2 + H^2}$ $k_{x_1} = k_{y_1} = 0.289\sqrt{3R^2 + 4H^2}$ $k_z = 0.707\ R$	$k_x = k_y = 0.289\sqrt{3(R^2 + r^2) + H^2}$ $k_z = 0.707\sqrt{R^2 + r^2}$ $k_{x_1} = k_{y_1} = \sqrt{\frac{R^2 + r^2}{4} + \frac{H^2}{3}}$

Adapted from Weight Handbook, Vol. 1, Society of Aeronautical Weight Engineers, Inc.

RIGHT CIRCULAR CONE



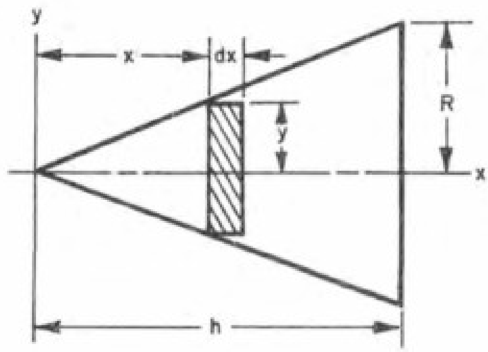
FRUSTUM OF A CONE



	RIGHT CIRCULAR CONE	FRUSTUM OF A CONE
VOLUME	$\frac{\pi R^2 H}{3}$	$\frac{\pi H}{3}(R^2 + Rr + r^2)$
CENTROID	$\bar{z} = \frac{H}{4}$	$\bar{z} = \frac{H}{4}\left[\frac{R^2 + 2Rr + 3r^2}{R^2 + Rr + r^2}\right]$
WEIGHT MOMENT OF INERTIA	$I_x = I_y = \frac{3W}{20}\left(R^2 + \frac{H^2}{4}\right)$ $I_{x_1} = I_{y_1} = \frac{W}{20}(3R^2 + 2H^2)$ $I_{x_2} = \frac{3W}{20}(R^2 + 4H^2)$ $I_z = \frac{3W}{10} R^2$	$I_z = \frac{3W}{10}\left[\frac{R^5 - r^5}{R^3 - r^3}\right]$
RADIUS OF GYRATION	$k_x = k_y = 0.387\sqrt{R^2 + \frac{H^2}{4}}$ $k_{x_1} = k_{y_1} = 0.224\sqrt{3R^2 + 2H^2}$ $k_{x_2} = 0.387\sqrt{R^2 + 4H^2}$ $k_z = 0.548\,R$	$k_z = 0.548\sqrt{\frac{R^5 - r^5}{R^3 - r^3}}$

Adapted from Weight Handbook, Vol. 1, Society of Aeronautical Weight Engineers, Inc.

CONE



By similar triangles, $y = (Rx/h)$

$$dm = \rho dV$$

$$I_x disk = mR^2/2$$

$$r^2 = y^2$$

$$dI_x = dmr^2/2$$

$$= (\rho\pi y^2 dx)(y^2/2)$$

$$I_x = \int r^2 dm$$

$$I_x = \int_0^h \rho\pi y^4 dx/2$$

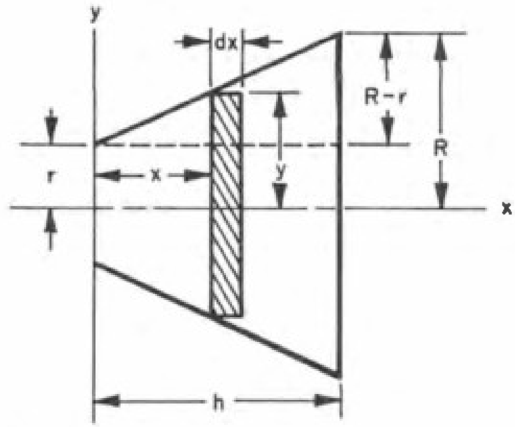
$$= \frac{\rho\pi}{2}\int_0^h \left(\frac{Rx}{h}\right)^4 dx = \frac{\rho\pi}{2}\left(\frac{R^4}{h^4}\right)\int_0^h x^4 dx$$

$$= \frac{\rho\pi R^4}{2h^4}\left[\frac{x^5}{5}\right]_0^h = \frac{\rho\pi R^4 h}{10}$$

$$\rho = \frac{m}{V} = \frac{m}{\pi R^2 h/3}$$

$$I_x = \frac{3m\pi R^4 h}{10\pi R^2 h} = \frac{3mR^2}{10} \text{ (ans.)}$$

FRUSTUM OF A CONE



By similar triangles,

$$\frac{R - r}{h} = \frac{y - r}{x}$$

$$x = \frac{hy - hr}{R - r} = \frac{hy}{R - r} - \frac{hr}{R - r}$$

$$dx = \left(\frac{h}{R - r}\right)dy$$

$$I_x disk = mR^2/2$$

$$dI_x = y^2 dm/2$$

$$dm = \rho dV = \rho \pi y^2 dx$$

$$dI_x = \frac{y^2}{2}(\rho \pi y^2 dx) = \frac{\rho \pi y^4}{2}\left(\frac{h}{R - r}\right)dy$$

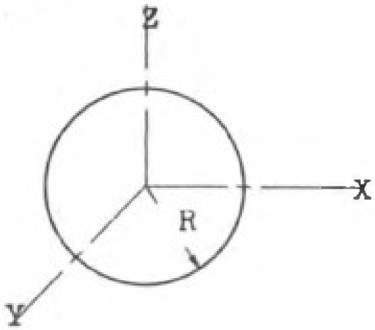
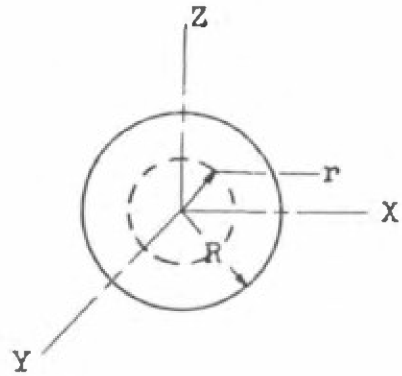
$$I_x = \frac{\rho \pi h}{2(R - r)} \int_r^R y^4 dy$$

$$= \frac{\rho \pi h}{2(R - r)}\left[\frac{y^5}{5}\right]_r^R = \frac{\rho \pi h}{10} \frac{(R^5 - r^5)}{(R - r)}$$

$$V = \frac{\pi h}{3} (R^2 + Rr + r^2)$$

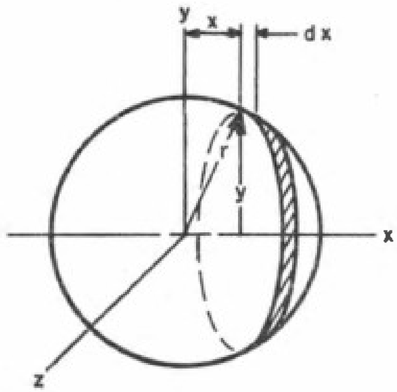
$$I_x = \frac{m \pi h (R^5 - r^5)(3)}{10 \pi h (R^2 + Rr + r^2)(R - r)}$$

$$= \frac{3m}{10} \frac{(R^5 - r^5)}{(R^3 - r^3)} \text{ (ans.)}$$

	SPHERE	HOLLOW SPHERE
		
VOLUME	$\frac{4}{3} \pi R^3$	$\frac{4}{3} \pi (R^3 - r^3)$
CENTROID	....	....
WEIGHT MOMENT OF INERTIA	$I_x = I_y = I_z = \frac{2}{5} WR^2$	$I_x = I_y = I_z = \frac{2}{5} W \left(\frac{R^5 - r^5}{R^3 - r^3}\right)$
RADIUS OF GYRATION	$k_x = k_y = k_z = 0.632 R$	$k_x = k_y = k_z = 0.632\sqrt{\left(\frac{R^5 - r^5}{R^3 - r^3}\right)}$

Adapted from Weight Handbook, Vol. 1, Society of Aeronautical Weight Engineers, Inc.

SOLID HOMOGENEOUS SPHERE



$$x = \sqrt{r^2 - y^2}$$

$$x^2 = r^2 - y^2$$

$$y^2 = r^2 - x^2$$

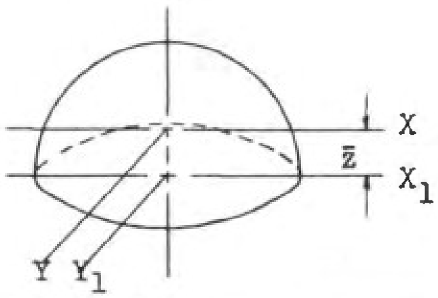
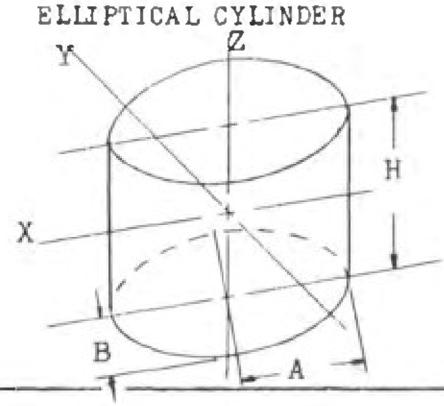
$$dm = \rho\pi y^2 dx$$

$$dI_x = \frac{dmy^2}{2} = \frac{(\rho\pi y^2 dx)y^2}{2} = \frac{\pi\rho(r^2 - x^2)^2 dx}{2}$$

$$I_x = \frac{\pi\rho}{2}\int_{-r}^{r} (r^2 - x^2)^2 dx = \frac{8\pi\rho r^5}{15}$$

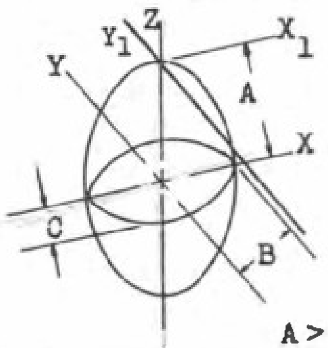
$$\rho = \frac{m}{V} = \frac{m}{(4/3)\pi r^3}$$

$$I_x = \frac{(8\pi r^5/15)m}{(4/3)\pi r^3} = \frac{2}{5}mr^2 \text{(ans.)}$$

	HEMISPHERE	ELLIPTICAL CYLINDER
		
VOLUME	$\frac{2}{3}\pi R^3$	πABH
CENTROID	$\bar{z} = \frac{3}{8} R$	$\bar{z} = \frac{H}{2}$
WEIGHT MOMENT OF INERTIA	$I_x = I_y = 0.26\ WR^2$ $I_{x_1} = I_{y_1} = I_z = 0.4WR^2$	$I_x = \frac{W}{12}(3B^2 + H^2)$ $I_y = \frac{W}{12}(3A^2 + H^2)$ $I_z = \frac{W}{4}(A^2 + B^2)$
RADIUS OF GYRATION	$k_x = k_y = 0.51R$ $k_{x_1} = k_{y_1} = k_z = 0.632R$	$k_x = 0.289\sqrt{3B^2 + H^2}$ $k_y = 0.289\sqrt{3A^2 + H^2}$ $k_z = \sqrt{\frac{A^2 + B^2}{2}}$

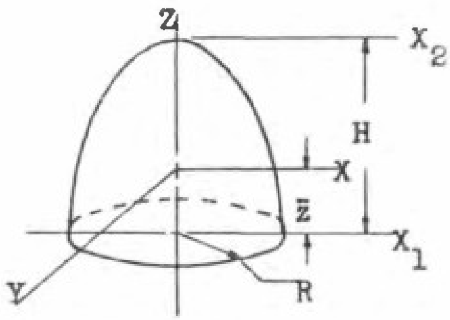
Adapted from Weight Handbook, Vol. 1, Society of Aeronautical Weight Engineers, Inc.

ELLIPSOID



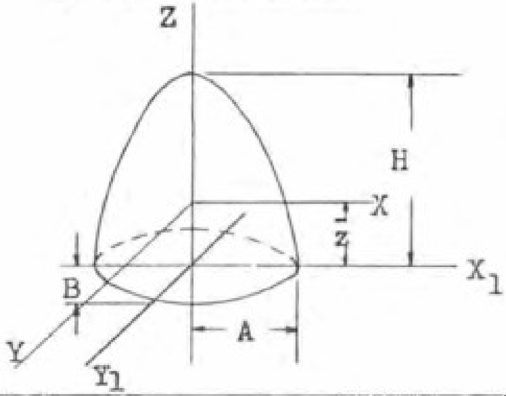
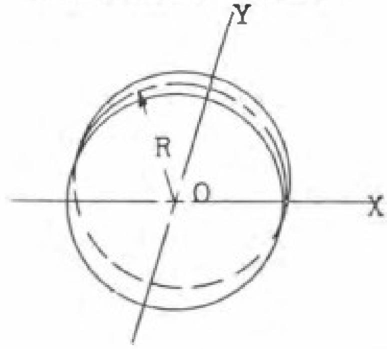
A > B > C

PARABOLOID OF REVOLUTION

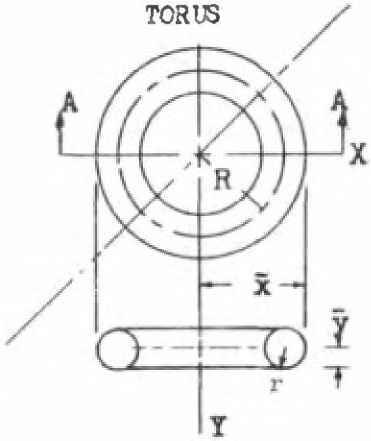
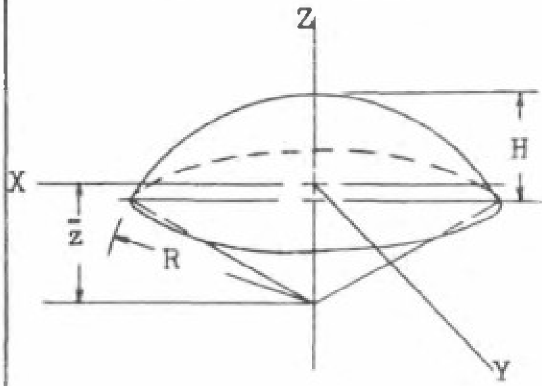


	ELLIPSOID	PARABOLOID OF REVOLUTION
VOLUME	$\frac{4}{3}\pi ABC$	$\frac{\pi R^2 H}{2}$
CENTROID	....	$\bar{z} = \frac{H}{3}$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{W}{5}(A^2 + C^2)$ $I_{x1} = \frac{W}{5}(6A^2 + C^2)$ $I_y = \frac{W}{5}(A^2 + B^2)$ $I_{y1} = \frac{W}{5}(6A^2 + B^2)$ $I_z = \frac{W}{5}(B^2 + C^2)$	$I_x = I_y = \frac{W}{18}(3R^2 + H^2)$ $I_{x_1} = \frac{W}{6}(R^2 + H^2)$ $I_{x_2} = \frac{W}{6}(R^2 + 3H^2)$ $I_z = \frac{WR^2}{3}$
RADIUS OF GYRATION	$k_x = 0.447\sqrt{A^2 + C^2}$ $k_{x_1} = 0.447\sqrt{6A^2 + C^2}$ $k_y = 0.447\sqrt{A^2 + B^2}$ $k_{y_1} = 0.447\sqrt{6A^2 + B^2}$ $k_z = 0.447\sqrt{B^2 + C^2}$	$k_x = k_y = 0.236\sqrt{3R^2 + H^2}$ $k_{x_1} = 0.408\sqrt{R^2 + H^2}$ $k_{x_2} = 0.408\sqrt{R^2 + 3H^2}$ $k_z = 0.577\ R$
SURFACE AREA *Approximate for prolate spheroid* → EXACT *for oblate spheroid* → *EXACT for prolate spheroid* →	Ellipsoid of revolution ~~or spheroid~~ $B = C$ $A = \frac{4\pi B}{\sqrt{2}}\sqrt{A^2 + B^2}$ $A = 2\pi A^2 + \pi\frac{B^2}{e}\log_e\frac{1 + e}{1 - e}$ $e = \text{eccentricity} = \frac{\sqrt{A^2 - B^2}}{A}$ *$A = 2\pi B^2 + 2\pi\frac{AB}{e}\sin^{-1} e$*	$A = \frac{2}{3}\pi R^2\left[m + \frac{1}{m + 1}\right]$ $m = \sqrt{1 + \left(\frac{2H}{R}\right)^2}$ *L.R. Kruse*

Adapted from Weight Handbook, Vol. 1, Society of Aeronautical Weight Engineers, Inc.

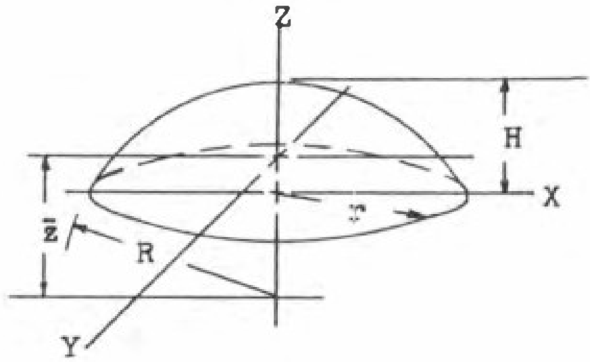
	ELLIPTIC PARABOLOID	THIN CIRCULAR LAMINA
		
VOLUME	$\frac{\pi ABH}{2}$	$V = \pi R^2 H$
CENTROID	$\bar{z} = \frac{H}{3}$	AT O = GEOMETRICAL CENTER
WEIGHT MOMENT OF INERTIA	$I_x = \frac{W}{18}(3B^2 + H^2)$ $I_{x1} = \frac{W}{6}(B^2 + H^2)$ $I_y = \frac{W}{18}(3A^2 + H^2)$ $I_{y1} = \frac{W}{6}(A^2 + H^2)$ $I_z = \frac{W}{6}(A^2 + B^2)$	$I_x = \frac{WR^2}{4}$ $I_y = \frac{WR^2}{2}$
RADIUS OF GYRATION	$k_x = 0.236\sqrt{3B^2 + H^2}$ $k_{x_1} = 0.408\sqrt{B^2 + H^2}$ $k_y = 0.236\sqrt{3A^2 + H^2}$ $k_{y_1} = 0.408\sqrt{A^2 + H^2}$ $k_z = 0.408\sqrt{A^2 + B^2}$	$k_x = \frac{R}{2}$ $k_y = \frac{R}{\sqrt{2}}$
NOTE	....	Y AXIS IS PERPENDICULAR TO PLANE H = THICKNESS

Adapted from Weight Handbook, Vol. 1, Society of Aeronautical Weight Engineers, Inc.

	TORUS	SPHERICAL SECTOR
		
VOLUME	$2\pi^2 r^2 R$	$\frac{2}{3} \pi R^2 H$
CENTROID	$\bar{x} = \bar{z} = R + r$ $\quad \bar{y} = r$	$\bar{z} = \frac{3}{8} (2R-H)$
WEIGHT MOMENT OF INERTIA	$I_x = I_z = \frac{W}{8} (4R^2 + 5r^2)$ $I_y = \frac{W}{4} (4R^2 + 3r^2)$	$I_z = \frac{WH}{5} (3R-H)$
RADIUS OF GYRATION	$k_x = k_z = 0.354 \sqrt{4R^2 + 5r^2}$ $k_y = \sqrt{\frac{4R^2 + 3r^2}{2}}$	$k_z = 0.447 \sqrt{(3R-H)(H)}$

Adapted from Weight Handbook, Vol. 1, Society of Aeronautical Weight Engineers, Inc.

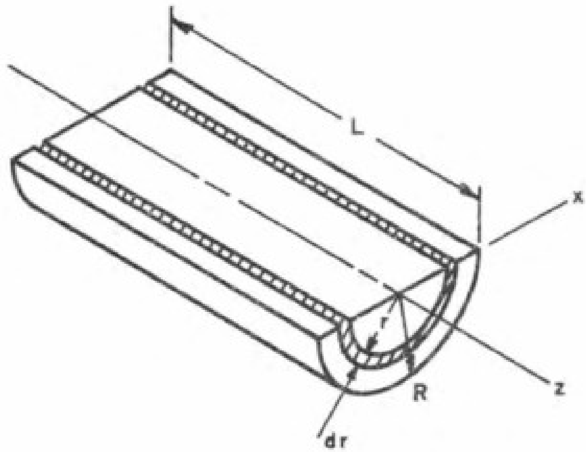
SPHERICAL SEGMENT



VOLUME	$\frac{\pi H}{6}(3r^2 + H^2) = \frac{\pi H^2}{3}(3R - H)$
CENTROID	$\bar{z} = \frac{3}{4}\frac{(2R - H)^2}{(3R - H)}$
WEIGHT MOMENT OF INERTIA	$I_z = \frac{2}{3}\frac{HW}{R-H}\left(R^2 - \frac{3}{4}RH + \frac{3}{20}H^2\right)$
RADIUS OF GYRATION	$k_z = \sqrt{\frac{2H}{3R-H}\left(R^2 - \frac{3}{4}RH + \frac{3}{20}H^2\right)}$

Adapted from Weight Handbook, Vol. 1, Society of Aeronautical Weight Engineers, Inc.

SEMICYLINDER



Elemental volume, half cylinder:

$$dm = \rho dV = \frac{\rho 2\pi r L dr}{2} = \rho \pi r L dr$$

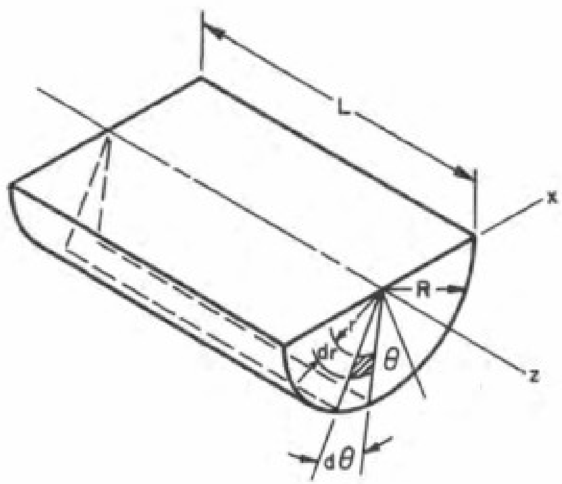
$$I_z = \int r^2 dm = \int_0^R r^2 (\rho \pi r L dr)$$

$$= \rho \pi L \int_0^R r^3 dr = \rho \pi L \left[\frac{r^4}{4}\right]_0^R = \frac{\rho \pi R^4 L}{4}$$

$$\rho = \frac{m}{V} = \frac{m}{\pi R^2 L/2}$$

$$I_z = \left(\frac{m}{\pi R^2 L/2}\right)\left(\frac{\pi R^4 L}{4}\right) = \frac{mR^2}{2} \text{ (ans.)}$$

SEMICYLINDER



Elemental volume, wedge shape:

$$dm = \rho dV = \rho r d\theta dr L$$

$$I_z = \int r^2 dm = \int_0^R \int_0^\pi r^2 (\rho r L d\theta dr)$$

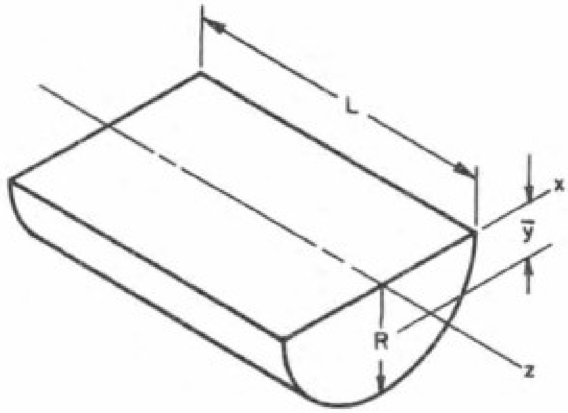
$$= \rho L \int_0^R \int_0^\pi r^3 d\theta dr = \rho L \pi \int_0^R r^3 dr$$

$$= \rho L \pi \left[\frac{r^4}{4}\right]_0^R = \frac{\rho L \pi R^4}{4}$$

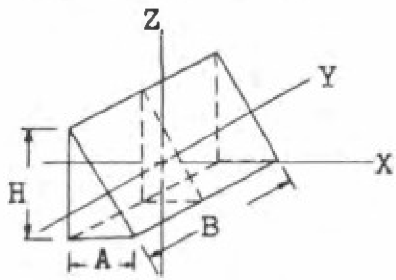
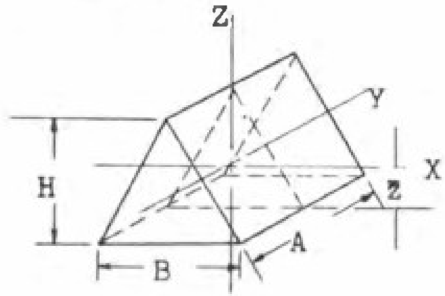
$$\rho = \frac{m}{V} = \frac{m}{\pi R^2 L/2}$$

$$I_z = \left(\frac{\pi L R^4}{4}\right)\left(\frac{2m}{\pi R^2 L}\right) = \frac{mR^2}{2} \quad \text{(ans.)}$$

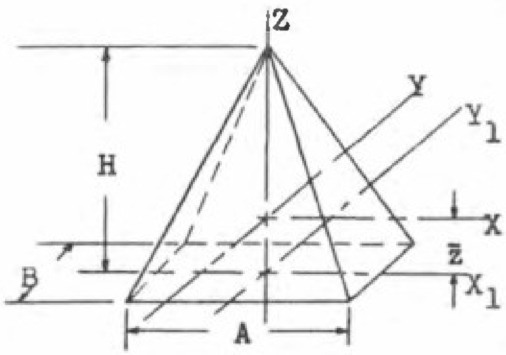
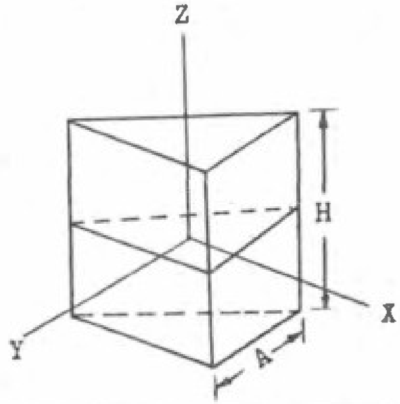
SEMICYLINDER



VOLUME	$\frac{\pi R^2 L}{2}$
CENTROID	$\bar{y} = \frac{4R}{3\pi}$
WEIGHT MOMENT OF INERTIA	$I_z = \frac{mR^2}{2}$
RADIUS OF GYRATION	$k_z = \frac{R}{\sqrt{2}} = 0.707R$

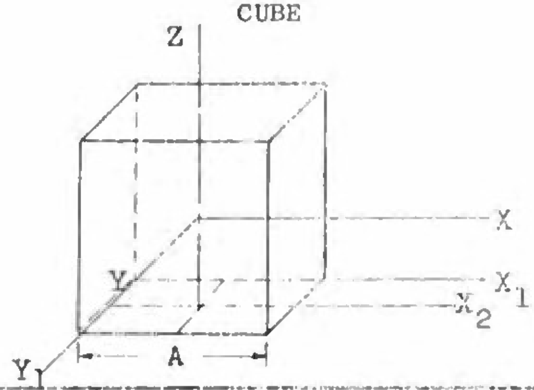
	RIGHT ANGLED WEDGE	ISOSCELES WEDGE
		
VOLUME	$\frac{ABH}{2}$	$\frac{ABH}{2}$
CENTROID	$\bar{x} = \frac{A}{3}$ $\bar{y} = \frac{B}{2}$ $\bar{z} = \frac{H}{3}$	$\bar{z} = \frac{H}{3}$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{W}{36}(2H^2 + 3B^2)$ $I_y = \frac{W}{18}(A^2 + H^2)$ $I_z = \frac{W}{36}(2A^2 + 3B^2)$	$I_x = \frac{W}{36}(2H^2 + 3A^2)$ $I_y = \frac{W}{72}(4H^2 + 3B^2)$ $I_z = \frac{W}{24}(2A^2 + B^2)$
RADIUS OF GYRATION	$k_x = 0.167\sqrt{2H^2 + 3B^2}$ $k_y = 0.236\sqrt{A^2 + H^2}$ $k_z = 0.167\sqrt{2A^2 + 3B^2}$	$k_x = 0.167\sqrt{2H^2 + 3A^2}$ $k_y = 0.118\sqrt{4H^2 + 3B^2}$ $k_z = 0.204\sqrt{2A^2 + B^2}$

Adapted from Weight Handbook, Vol. 1, Society of Aeronautical Weight Engineers, Inc.

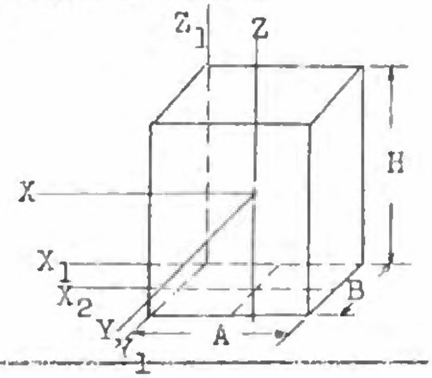
	RIGHT RECTANGULAR PYRAMID	REGULAR TRIANGULAR PRISM
		
VOLUME	$\frac{ABH}{3}$	$\frac{\sqrt{3}}{4} A^2H$
CENTROID	$\bar{z} = \frac{H}{4}$	$\bar{z} = \frac{H}{2}$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{W}{20}(B^2 + \frac{3H^2}{4})$ $I_{x_1} = \frac{W}{20}(B^2 + 2H^2)$ $I_y = \frac{W}{20}(A^2 + \frac{3}{4}H^2)$ $I_{y_1} = \frac{W}{20}(A^2 + 2H^2)$ $I_z = \frac{W}{20}(A^2 + B^2)$	$I_x = I_y = \frac{W}{24}(A^2 + 2H^2)$ $I_z = \frac{WA^2}{12}$
RADIUS OF GYRATION	$k_x = 0.224\sqrt{B^2 + \frac{3H^2}{4}}$ $k_{x_1} = 0.224\sqrt{B^2 + 2H^2}$ $k_y = 0.224\sqrt{A^2 + \frac{3H^2}{4}}$ $k_{y_1} = 0.224\sqrt{A^2 + 2H^2}$ $k_z = 0.224\sqrt{A^2 + B^2}$	$k_x = k_y = 0.204\sqrt{A^2 + 2H^2}$ $k_z = 0.289A$

Adapted from Weight Handbook, Vol. I, Society of Aeronautical Weight Engineers, Inc.

CUBE



RECTANGULAR PRISM



	CUBE	RECTANGULAR PRISM
VOLUME	A^3	ABH
CENTROID	$\bar{x} = \bar{y} = \bar{z} = \frac{A}{2}$	$\bar{x} = \frac{A}{2}$ $\bar{y} = \frac{B}{2}$ $\bar{z} = \frac{H}{2}$
WEIGHT MOMENT OF INERTIA	$I_x = I_y = I_z = \frac{WA^2}{6}$ $I_{x_1} = I_{y1} = I_{z_1} = \frac{2WA^2}{3}$ $I_{x_2} = \frac{5WA^2}{12}$	$I_x = \frac{W}{12}(B^2 + H^2)$ $I_{x1} = \frac{W}{3}(B^2 + H^2)$ $I_{x2} = \frac{W}{12}(B^2 + 4H^2)$ $I_y = \frac{W}{12}(A^2 + H^2)$ $I_{y1} = \frac{W}{3}(A^2 + H^2)$ $I_z = \frac{W}{12}(A^2 + B^2)$ $I_{z1} = \frac{W}{3}(A^2 + B^2)$
RADIUS OF GYRATION	$k_x = k_y = k_z = 0.408A$ $k_{x_1} = k_{y_1} = k_{z_1} = 0.816A$ $k_{x2} = 0.646A$	$k_x = 0.289\sqrt{B^2 + H^2}$ $k_{x_1} = 0.577\sqrt{B^2 + H^2}$ $k_{x_2} = 0.289\sqrt{B^2 + 4H^2}$ $k_y = 0.289\sqrt{A^2 + H^2}$ $k_{y_1} = 0.577\sqrt{A^2 + H^2}$ $k_z = 0.289\sqrt{A^2 + B^2}$ $k_{z_1} = 0.577\sqrt{A^2 + B^2}$

Adapted from Weight Handbook, Vol. 1, Society of Aeronautical Weight Engineers, Inc.

THIN SHELLS

A thin shell can be developed by the subtraction of a smaller inner solid from a larger outer solid of similar shape, by the summation of elemental rings by integration, or by revolving an arc or a segment of an arc about a desired coordinate axis.

SUMMARY OF EQUATIONS

Surface Area Generated by Revolving Arc

If A(a,c) and B(b,d) are two points on a curve $F(x,y) = 0$. The area of the surface generated by revolving the arc AB about the x-axis is given by

$$S = 2\pi\int_{AB} y\,ds = 2\pi\int_a^b y\sqrt{1 + (dy/dx)^2}\,dx$$

or

$$2\pi\int_c^d y\sqrt{1 + (dy/dx)^2}\,dy$$

When revolved about the y-axis, the arc AB generates a surface area

$$S = 2\pi\int_{AB} x\,ds = 2\pi\int_a^b x\sqrt{1 + (dy/dx)^2}\,dx$$

or

$$2\pi\int_c^d x\sqrt{1 + (dx/dy)^2}\,dy$$

If A, given by $u = u_1$, and B, given by $u = u_2$, are two points on a curve that is defined by the parametric equations $x = f(u)$, $y = g(u)$, the surface area generated by revolving the arc AB about the x-axis is

$$S = 2\pi\int_{AB} y\,ds = 2\pi\int_{u_1}^{u_2} y\sqrt{(dx/du)^2 + (dy/du)^2}\,du$$

and the area generated about the y-axis bounded by AB is

$$S = 2\pi\int_{AB} x\,ds = 2\pi\int_{u_1}^{u_2} x\sqrt{(dx/du)^2 + (dy/du)^2}\,du$$

Centroid of a Surface of Revolution

The centroid of a surface of revolution generated by the arc of a curve $y = f(x)$ extending from A(a,c) to B(b,d) is defined by

$$S\bar{x} = 2\pi\int_a^b xy\,ds$$

where ds is an element of arc as described in the above equations as

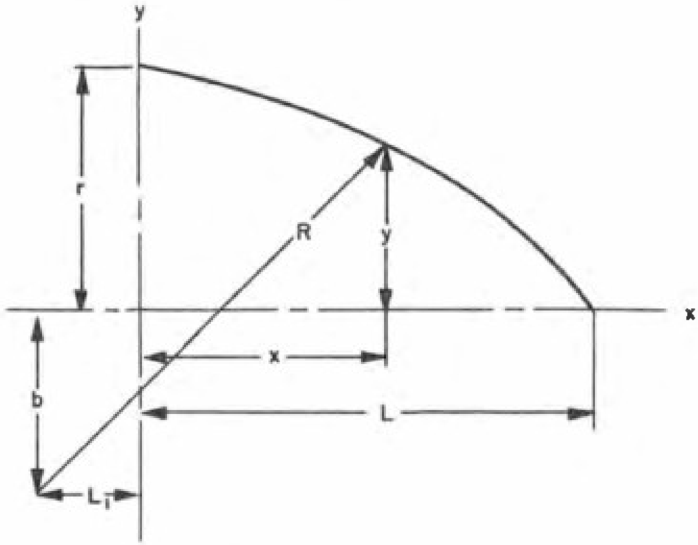
$$ds = \sqrt{1 + (dy/dx)^2}\,dx$$

when taken about the x-axis, and

$$S\bar{y} = 2\pi\int_a^b xy\,ds$$

when taken about the y-axis.

SURFACE AREA AND CENTROIDAL DISTANCE



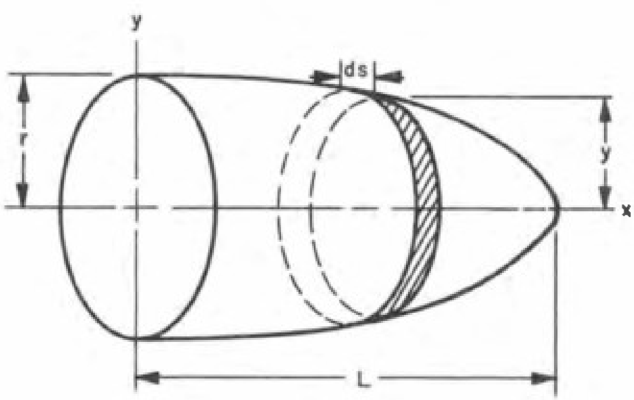
From the diagram above, the variable radius y is

$$(x + L_1)^2 + (y + b)^2 = R^2$$

$$y + b = \sqrt{R^2 - (x + L_1)^2}$$

$$y = \sqrt{R^2 - (x + L_1)^2} - b$$

$$= \left[(R^2 - L_1^2) - 2L_1x - x^2\right]^{1/2} - b$$



The surface area developed is

$$S = \int 2\pi y ds \qquad \text{where} \qquad ds = \sqrt{1 + (dy/dx)^2}dx$$

$$\frac{dy}{dx} = \left(\frac{1}{2}\right)\frac{-2x - 2L_1}{\left[(R^2 - L_1^2) - 2L_1x - x^2\right]^{1/2}} = -\frac{x + L_1}{(R^2 - L_1^2 - 2L_1x - x^2)^{1/2}}$$

$$1 + \left(\frac{dy}{dx}\right)^2 = \frac{(x + L_1)^2}{R^2 - L_1^2 - 2L_1x - x^2} + 1 = \frac{R^2}{R^2 - L_1^2 - 2L_1x - x^2}$$

$$ds = \frac{R\ dx}{(R^2 - L_1^2 - 2L_1x - x^2)^{\frac{1}{2}}}$$

$$S = 2\pi\int_0^L \left\{ \left[(R^2 - L_1^2) - 2L_1x - x^2 \right]^{1/2} - b \right\} \left(\frac{R^2}{R^2 - L_1^2 - 2L_1x - x^2} \right)^{1/2} dx$$

$$S = 2\pi R \int_0^L \left[1 - \frac{b}{(R^2 - L_1^2 - 2L_1x - x^2)^{1/2}} \right] dx$$

$$= 2\pi R \left[x - b \sin^{-1} \frac{x + L_1}{R} \right]_0^L$$

$$= 2\pi R \left[L - b \left(\sin^{-1} \frac{L + L_1}{R} - \sin^{-1} \frac{L_1}{R} \right) \right] \quad \text{(ans.)}$$

The centroid of the shell can be determined by applying the basic mathematical statement

$$S\bar{x} = 2\pi\int_0^L xyds \qquad \text{where} \qquad ds = \sqrt{1 + (dy/dx)^2}\,dx$$

Then

$$S\bar{x} = 2\pi\int_0^L xy\sqrt{1 + (dy/dx)^2}\,dx$$

and substituting $y = \left[(R^2 - L_1^2) - 2L_1x - x^2 \right]^{1/2} - b$,

$$S\bar{x} = 2\pi\int_0^L x\left\{ \left[(R^2 - L_1^2) - 2L_1x - x^2 \right]^{1/2} - b \right\} \left\{ \frac{R}{\left[(R^2 - L_1^2) - 2L_1x - x^2 \right]^{1/2}} \right\} dx$$

$$= 2\pi R\int_0^L \left\{ x - \frac{xb}{\left[(R^2 - L_1^2) - 2L_1x - x^2 \right]^{1/2}} \right\} dx$$

$$= 2\pi R \left[\frac{x^2}{2} + b \left\{ \left[(R^2 - L_1^2) - 2L_1x - x^2 \right]^{1/2} + L_1 \sin^{-1} \frac{x + L_1}{R} \right\} \right]_0^L$$

$$= 2\pi R \left[\left(\frac{L^2}{2} + b \left\{ \left[(R^2 - L_1^2) - 2L_1L - L^2 \right]^{1/2} + L_1 \sin^{-1} \frac{L + L_1}{R} \right\} \right) - \left\{ b \left[(R^2 - L_1^2)^{1/2} + L_1 \sin^{-1} \frac{L_1}{R} \right] \right\} \right]$$

$$= 2\pi R \left\{ \left(\frac{L^2}{2} + b^2 + bL_1 \sin^{-1} \frac{L + L_1}{R} \right) - b \left[(b + r) + L_1 \sin^{-1} \frac{L_1}{R} \right] \right\}$$

$$= 2\pi R \left[\frac{L^2}{2} - br + bL_1 \left(\sin^{-1} \frac{L + L_1}{R} - \sin^{-1} \frac{L_1}{R} \right) \right]$$

$$\bar{x} = \frac{2\pi R\left((L^2/2) - br + bL_1 \left\{ \sin^{-1} \left[(L + L_1)/R \right] - \sin^{-1} (L_1/R) \right\} \right)}{2\pi R\left(L - b \left\{ \sin^{-1} \left[(L + L_1)/R \right] - \sin^{-1} (L_1/R) \right\} \right)}$$

$$= \frac{(L^2/2) - br + bL_1 \left\{ \sin^{-1} \left[(L + L_1)/R \right] - \sin^{-1} (L_1/R) \right\}}{L - b\left\{ \sin^{-1} \left[(L + L_1)/R \right] - \sin^{-1} (L_1/R) \right\}} \quad \text{(ans.)}$$

Applying this result to a thin-shelled hemisphere, it can be seen that all terms containing b reduce to zero, which leaves

$$\bar{x} = L/2, \text{ but } L = R \text{ for the hemisphere;}$$

therefore,

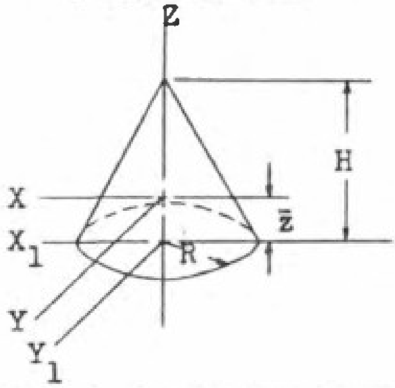
$$\bar{x} = R/2 \quad \text{(ans.)}$$

An alternative method for determining the location of the centroid of a thin-shelled ogive is to perform volume subtractions: that is, to calculate the volume of the ogive that corresponds to the exterior dimensions desired and then to subtract a volume of the proportions that will ultimately leave the desired wall thickness. Combining the centroidal distances of the two volumes with their respective values of volume, the basic equation becomes

$$\bar{x} = \frac{\Sigma V\bar{x}}{V} = \frac{(V\bar{x})_O - (V\bar{x})_I}{V_O - V_I}$$

where $(V\bar{x})_O$ and $(V\bar{x})_I$ are the statical moments of the outer and inner volumes, respectively, with V_O and V_I representing the outer and inner volumes.

LATERAL SURFACE OF
A CIRCULAR CONE

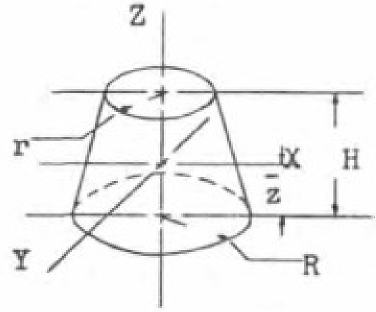


CONSIDER I_x AS THE SUM OF TWO MOMENTS OF INERTIA

CIRCLE OF RADIUS R

TRIANGLE OF ALTITUDE H
WT. OF CIRCLE AND TRIANGLE EQUALS WT. OF SURFACE

LATERAL SURFACE OF
FRUSTUM OF
CIRCULAR CONE



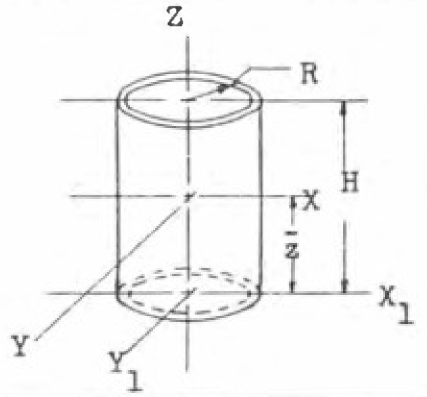
CONSIDER I_x AS THE SUM OF TWO MOMENTS OF INERTIA

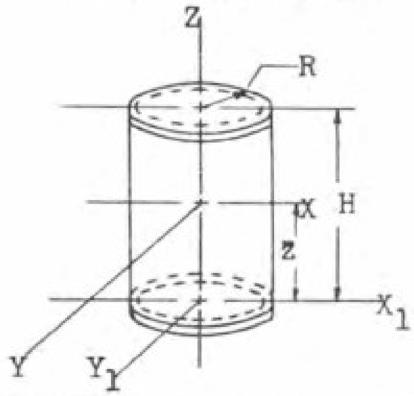
CIRCLE OF RADIUS R

TRIANGLE OF ALTITUDE H
WT. OF CIRCLE AND TRIANGLE EQUALS WT. OF SURFACE

	LATERAL SURFACE OF A CIRCULAR CONE	LATERAL SURFACE OF FRUSTUM OF CIRCULAR CONE
SURFACE AREA	$\pi R \sqrt{R^2 + H^2}$	$\pi (R + r) \sqrt{H^2 + (R - r)^2}$
CENTROID	$\bar{z} = \frac{H}{3}$	$\bar{z} = \frac{H}{3} \left\{\frac{2r + R}{r + R}\right\}$
WEIGHT MOMENT OF INERTIA	$I_x = I_y = \frac{W}{4} (R^2 + \frac{2}{9} H^2)$ $I_z = \frac{WR^2}{2}$ $I_{x_1} = I_{y_1} = \frac{W}{12} (3R^2 + 2H^2)$	$I_x = I_y = \frac{W}{4} (R^2 + r^2) + \frac{WH^2}{18} \left\{1 + \frac{2Rr}{(R+r)^2}\right\}$ $I_z = \frac{W}{2} (R^2 + r^2)$
RADIUS OF GYRATION	$k_x = k_y = \sqrt{\frac{9R^2 + 2H^2}{6}}$ $k_{x_1} = k_{y_1} = 0.289 \sqrt{3R^2 + 2H^2}$ $k_z = 0.707 R$	$k_x = k_y = \sqrt{\frac{(R^2+r^2)}{4} + \frac{H^2}{18} \left\{1 + \frac{2Rr}{(R+r)^2}\right\}}$ $k_z = 0.707 \sqrt{R^2 + r^2}$

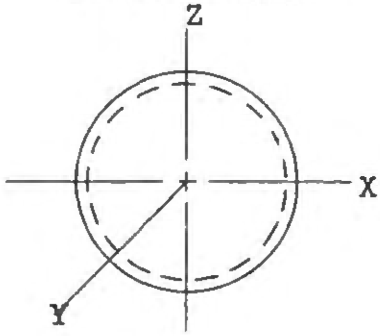
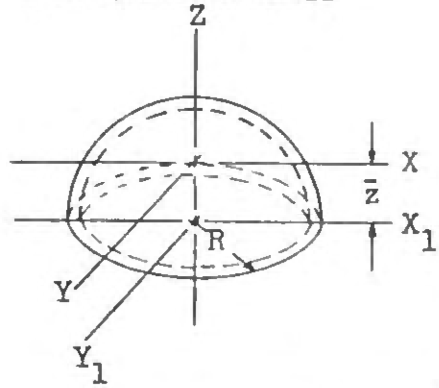
Adapted from Weight Handbook, Vol. 1, Society of Aeronautical Weight Engineers, Inc.





	LATERAL CYLINDRICAL SHELL	TOTAL CYLINDRICAL SHELL
SURFACE AREA	$2\pi RH$	$2\pi R(R+H)$
CENTROID	$\bar{z} = \frac{H}{2}$	$\bar{z} = \frac{H}{2}$
WEIGHT MOMENT OF INERTIA	$I_x = I_y = \frac{W}{2}\left(R^2 + \frac{H^2}{6}\right)$ $I_z = WR^2$ $I_{x_1} = I_{y_1} = \frac{W}{6}(3R^2 + 2H^2)$	$I_x = I_y = \frac{W}{12(R+H)}\left[3R^2(R+2H) + H^2(3R+H)\right]$ $I_{x_1} = I_{y_1} = \frac{W}{12(R+H)}\left[3R^2(R+2H) + 2H^2(3R+2H)\right]$ $I_z = \frac{WR^2}{2}\left[\frac{R+2H}{R+H}\right]$
RADIUS OF GYRATION	$k_x = k_y = 0.707\sqrt{R^2 + \frac{H^2}{6}}$ $k_{x_1} = k_{y_1} = 0.408\sqrt{3R^2 + 2H^2}$ $k_z = R$	$k_x = k_y = 0.289\sqrt{\frac{3R^2(R+2H) + H^2(3R+H)}{R+H}}$ $k_{x_1} = k_{y_1} = 0.289\sqrt{\frac{3R^2(R+2H) + 2H^2(3R+2H)}{R+H}}$ $k_z = 0.707\,R\sqrt{\frac{R+2H}{R+H}}$

Adapted from Weight Handbook, Vol. 1, Society of Aeronautical Weight Engineers, Inc.

	SPHERICAL SHELL	HEMISPHERICAL SHELL
		
SURFACE AREA	$4 \pi R^2$	$2 \pi R^2$
CENTROID	$\bar{x} = \bar{y} = \bar{z} = 0$	$\bar{z} = \frac{R}{2}$
WEIGHT MOMENT OF INERTIA	$I_x = I_y = I_z = \frac{2}{3} WR^2$	$I_x = I_y = \frac{5}{12} WR^2$ $I_z = I_{x_1} = I_{y_1} = \frac{2}{3} WR^2$
RADIUS OF GYRATION	$k_x = k_y = k_z = 0.816\ R$	$k_x = k_y = 0.646\ R$ $k_z = k_{x_1} = k_{y_1} = 0.816\ R$

Adapted from Weight Handbook, Vol. 1, Society of Aeronautical Weight Engineers, Inc.

THIN RODS

A thin rod, or wire, is so designated when the length, L, is greater than 30 times the diameter, d. Lengths of curved arcs will be designated by s as noted.

SUMMARY OF EQUATIONS

Length of Arc

The length of arc of the curve $F(x,y) = 0$ limited by two selected points $A(a,c)$ and $B(b,d)$ is given by

$$s = \int_{AB} ds = \int_a^b \sqrt{1 + (dy/dx)^2}\,dx \qquad \text{or} \qquad \int_c^d \sqrt{1 + (dx/dy)^2}\,dy$$

Example. For the length of arc of the curve $y = x^{3/2}$ from $x = 0$ to $x = 5$,

$$\frac{dy}{dx} = \frac{3}{2}x^{1/2} \qquad \text{and} \qquad 1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{9}{4}x$$

then

$$s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx = \int_0^5 \sqrt{1 + \frac{9}{4}x}\,dx = \frac{8}{27}\left(1 + \frac{9}{4}x\right)^{3/2}\Bigg]_0^5 = \frac{335}{27} \text{ (ans.)}$$

If A, given by $u = u_1$, and B, given by $u = u_2$, are points on a curve defined by the parametric equations $x = f(u)$, $y = g(u)$, the length of arc AB is given by

$$s = \int_{AB} ds = \int_{u_1}^{u_2} \sqrt{(dx/du)^2 + (dy/du)^2}\,du$$

Example. For the length of arc of the curve $x = t^2$, $y = t^3$ from $t = 0$ to $t = 4$,

$$\frac{dx}{dt} = 2t, \qquad \frac{dy}{dt} = 3t^2$$

and

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 4t^2 + 9t^4 = 4t^2\left(1 + \frac{9}{4}t^2\right)$$

then

$$s = \int_0^4 \sqrt{1 + \frac{9}{4}t^2}\,(2t\,dt) = \frac{8}{27}\left(1 + \frac{9}{4}t^2\right)^{3/2}\Bigg]_0^4 = \frac{8}{27}(37\sqrt{37} - 1) \text{ (ans.)}$$

Centroid of an Arc

The centroidal coordinates $(\bar{x},\bar{y})$ of an arc of a plane curve of equation $F(x,y) = 0$ or $x = f(u)$, $y = g(u)$ can be determined by the relationships

$$\bar{x}s = \bar{x}\int ds = \int x\,ds \qquad \text{and} \qquad \bar{y}s = \bar{y}\int ds = \int y\,ds$$

where the limits of integration are determined from the extent of the desired integration.

Example. For the centroid of the first quadrant arc of the circle $x^2 + y^2 = 25$,

$$\frac{dy}{dx} = -\frac{x}{y} \qquad \text{and} \qquad 1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{x^2}{y^2} = \frac{25}{y^2}$$

since $s = R\theta = 5\pi/2$

$$\frac{5\pi\bar{y}}{2} = \int_0^5 y\sqrt{1 + (dy/dx)^2}\,dx = \int_0^5 5\,dx = 25$$

and $\bar{y} = 10/\pi$ by symmetry, $\bar{x} = \bar{y}$, and the coordinates of the centroid are

$$\left(\frac{10}{\pi},\frac{10}{\pi}\right) \text{ (ans.)}$$

Moments of Inertia of an Arc

The moments of inertia of an arc, referred to the coordinate axes, are given by

$$I_x = \int_\alpha^\beta y^2\,ds \qquad \text{and} \qquad I_y = \int_\alpha^\beta x^2\,ds$$

Example. For the moment of inertia of the arc of a circle with respect to a fixed diameter,

$$\frac{dy}{dx} = -\frac{x}{y}, \qquad \sqrt{1 + (dy/dx)^2} = R/y, \qquad s = 2\pi R$$

The total moment of inertia is four times that of the first quadrant arc.

$$I_x = 4\int_0^R y^2\,ds = 4\int_0^R y^2\frac{R}{y}dx = 4R\int_0^R \sqrt{R^2 - x^2}\,dx = \pi R^3 = \frac{R^2 s}{2} \text{ (ans.)}$$

It should be noted that the value obtained contains only linear dimensions; therefore, to obtain the moment of inertia, I_x, in terms of mass, $m = \rho s$, the transformation should be made as $s = 2\pi R$. Then

$$I_x = \rho\pi R^3 \qquad \text{and} \qquad \rho = m/2\pi R$$

which results in

$$I_x = mR^2/2 \quad \text{(ans.)}$$

Centroid of an Area

The theorem of Pappus for the determination of the centroid of an area produced by the revolution of an arc is as follows.

If an arc of a curve is revolved about an axis in its plane and not crossing the arc, the area of the surface generated is equal to the product of the length of arc and the length of the path described by the centroid of the arc.

Example. For the centroid of the first quadrant arc of a circle of radius R,

$$s = 2\pi R^2 = (1/2\pi R)(2\pi \bar{x})$$

by symmetry, $\bar{x} = \bar{y}$, and the centroid has coordinates $(2R/\pi, 2R/\pi)$.

It follows that the appropriate surface area can be found from the same theorem if the centroidal distance is known.

Moment of Inertia of a Thin Rod

The moment of inertia of a long, slender rod can be calculated from the relationships

$$I_y = \int_0^L x^2 dm \qquad \text{or} \qquad I_x = \int_0^L y^2 dm$$

depending on the choice of axes.

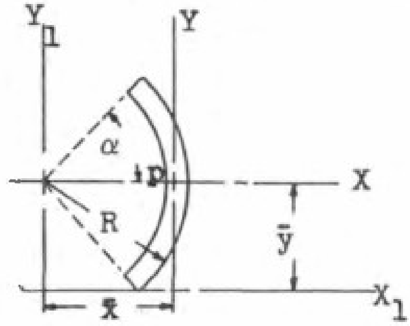
Example. For the moment of inertia of a thin homogeneous rod about an end,

$$I_y = \int_0^L x^2 dm = \rho\int_0^L x^2 dx = \rho L^3/3$$

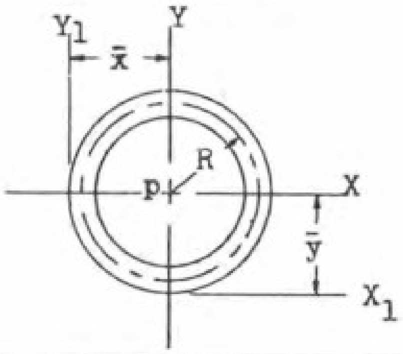
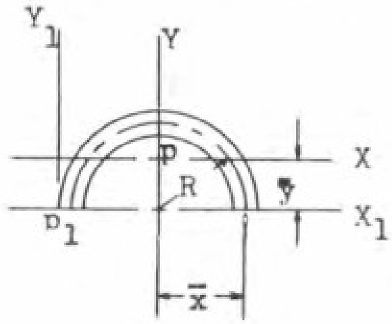
where $dm = \rho dx$

Recalling that $\rho = m/L$,

$$I_y = (m/L)(L^3/3) = mL^3/3 \quad \text{(ans.)}$$

	SEGMENT OF A CIRCULAR ROD
	α **in radians** 
LENGTH	$2R\alpha$
CENTROID	$\bar{x} = \frac{R \sin\alpha}{\alpha}$ $\bar{y} = R \sin\alpha$
WEIGHT MOMENT OF INERTIA	$I_x = WR^2 \left(\frac{1}{2} - \frac{\sin\alpha\cos\alpha}{2\alpha}\right)$ $I_{x_1} = WR^2 \left[\left(\frac{1}{2} - \frac{\sin\alpha\cos\alpha}{2\alpha}\right) + \sin^2\alpha\right]$ $I_y = WR^2 \left[\left(\frac{1}{2} + \frac{\sin\alpha\cos\alpha}{2\alpha}\right) - \frac{\sin^2\alpha}{\alpha^2}\right]$ $I_{y_1} = WR^2 \left(\frac{1}{2} + \frac{\sin\alpha\cos\alpha}{2\alpha}\right)$ $I_p = WR^2 \left(1 - \frac{\sin^2\alpha}{\alpha^2}\right)$
RADIUS OF GYRATION	$k_x = R\sqrt{\frac{1}{2} - \frac{\sin\alpha\cos\alpha}{2\alpha}}$ $k_{x_1} = R\sqrt{\frac{1}{2} - \frac{\sin\alpha\cos\alpha}{2\alpha} + \sin^2\alpha}$ $k_y = R\sqrt{\frac{1}{2} + \frac{\sin\alpha\cos\alpha}{2\alpha} - \frac{\sin^2\alpha}{\alpha^2}}$ $k_{y_1} = R\sqrt{\frac{1}{2} + \frac{\sin\alpha\cos\alpha}{2\alpha}}$ $k_p = R\sqrt{1 - \frac{\sin^2\alpha}{\alpha^2}}$

Adapted from Weight Handbook, Vol. 1, Society of Aeronautical Weight Engineers, Inc.

	CIRCULAR ROD 	SEMICIRCULAR ROD 
LENGTH	$2\pi R$	πR
CENTROID	$\bar{x} = \bar{y} = R$	$\bar{x} = R$ $\quad \bar{y} = 0.6366R$
WEIGHT MOMENT OF INERTIA	$I_x = I_y = \frac{WR^2}{2}$ $I_z = WR^2$ $I_{x_1} = I_{y_1} = \frac{3WR^2}{2}$ $I_p = WR^2$	$I_x = 0.0947\ WR^2$ $I_{x_1} = 0.5\ WR^2$ $I_y = 0.5\ WR^2$ $I_{y_1} = 1.5\ WR^2$ $I_p = 0.5947\ WR^2$ $I_{p_1} = 2\ WR^2$
RADIUS OF GYRATION	$k_x = k_y = 0.707\ R$ $k_{x_1} = k_{y_1} = 1.225\ R$ $k_p = R$	$k_x = 0.308\ R$ $k_{x_1} = 0.707\ R$ $k_y = 0.707\ R$ $k_{y_1} = 1.225\ R$ $k_p = 0.771\ R$ $k_{p_1} = 1.414\ R$

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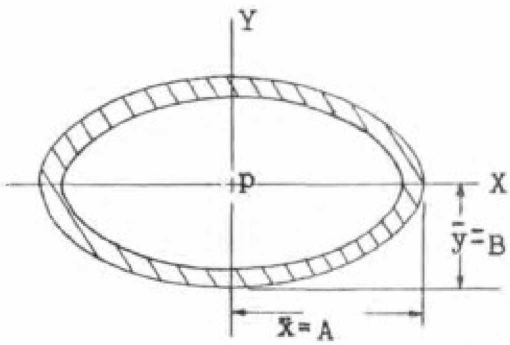
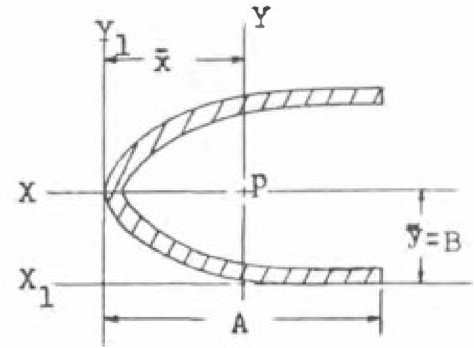
	ELLIPTIC ROD	PARABOLIC ROD
		
LENGTH	$\frac{\pi}{2A}(3A^2+B^2)$ (More Accurate) $\frac{\pi}{32A^3}\left[45A^4+22A^2B^2-3B^4\right]$	$L=\sqrt{4A^2+B^2}+\frac{B^2}{2A}\log_e\frac{2A+\sqrt{4A^2+B^2}}{B}$
CENTROID	$\bar{x}=A$ $\quad\bar{y}=B$	$\bar{x}=\frac{\sqrt{(4A^2+B^2)^3}}{8AL}-\frac{B^2}{16A}$ $\quad\bar{y}=B$
WEIGHT MOMENT OF INERTIA	$I_x=\frac{WB^2(55A^4+10A^2B^2-B^4)}{2(45A^4+22A^2B^2-3B^4)}$ $I_y=\frac{WA^2(35A^4+34A^2B^2-5B^4)}{2(45A^4+22A^2B^2-3B^4)}$ $I_p=I_x+I_y$	$I_x=\frac{WB^2}{8A^2}\left(\frac{\sqrt{(4A^2+B^2)^3}}{L}-\frac{B^2}{2}\right)$ $I_{x_1}=\frac{WB^2}{8A^2}\left(\frac{\sqrt{(4A^2+B^2)^3}}{L}-\frac{B^2}{2}\right)+WB^2$ $I_y=\frac{W\sqrt{(4A^2+B^2)^3}}{12L}-\frac{I_x}{8}-Wx^2$ $I_{y_1}=\frac{W\sqrt{(4A^2+B^2)^3}}{12L}-\frac{1}{8}I_x$ $I_p=I_x+I_y$
RADIUS OF GYRATION	$k_x=\sqrt{\frac{I_x}{W}}$ $k_y=\sqrt{\frac{I_y}{W}}$ $k_p=\sqrt{\frac{I_p}{W}}$	$k_x=\sqrt{\frac{I_x}{W}}$ $k_{x_1}=\sqrt{\frac{I_{x_1}}{W}}$ $k_y=\sqrt{\frac{I_y}{W}}$ $k_{y_1}=\sqrt{\frac{I_{y_1}}{W}}$ $k_p=\sqrt{\frac{I_p}{W}}$
NOTE	$A > B$	

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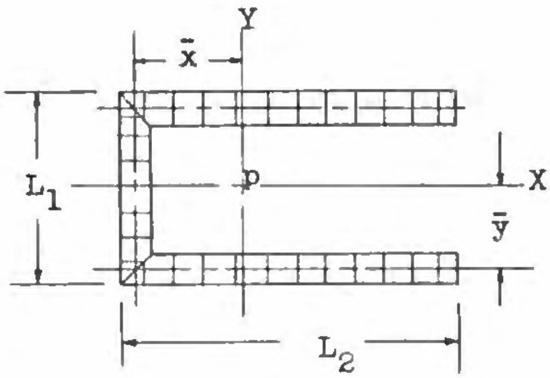
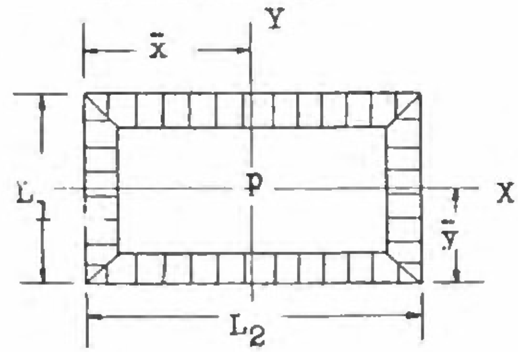
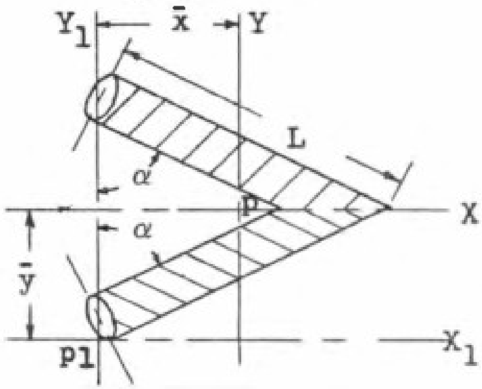
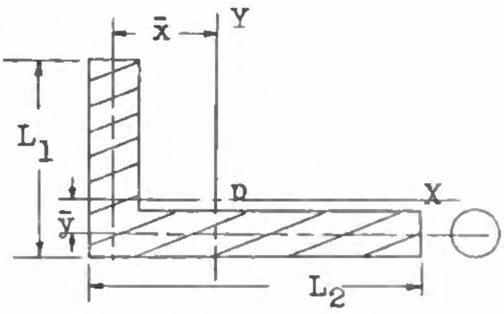
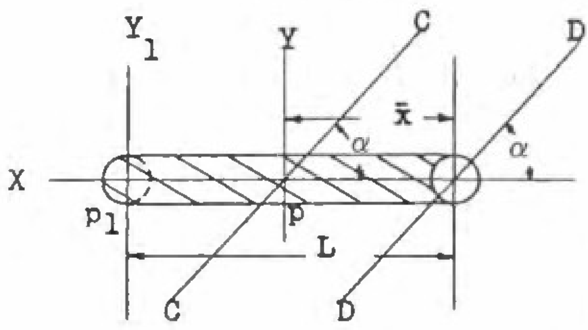
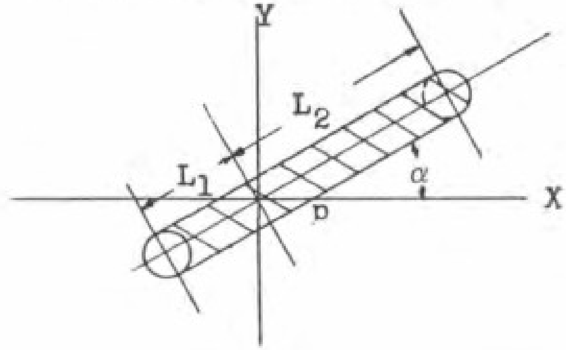
	U-ROD	RECTANGULAR ROD
		
LENGTH	$L_1 + 2L_2$	$2(L_1 + L_2)$
CENTROID	$\bar{x} = \frac{L_2^2}{L_1+2L_2}$ $\quad \bar{y} = \frac{L_1}{2}$	$\bar{x} = \frac{L_2}{2}$ $\quad \bar{y} = \frac{L_1}{2}$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{WL_1^2(L_1 + 6L_2)}{12(L_1 + 2L_2)}$ $I_y = \frac{WL_2^3\,(2L_1 + L_2)}{3\,(L_1+2L_2)^2}$ $I_p = I_x + I_y$	$I_x = \frac{WL_1^2(L_1+3L_2)}{12(L_1+L_2)}$ $I_y = \frac{WL_2^2(3L_1+L_2)}{12(L_1+L_2)}$ $I_p = I_x + I_y$
RADIUS OF GYRATION	$k_x = 0.289L_1\sqrt{\frac{L_1+6L_2}{L_1+2L_2}}$ $k_y = \frac{0.577L_2}{L_1+2L_2}\sqrt{L_2(2L_1+L_2)}$ $k_p = \sqrt{\frac{I_p}{W}}$	$k_x = 0.289L_1\sqrt{\frac{L_1+3L_2}{L_1+L_2}}$ $k_y = 0.289L_2\sqrt{\frac{3L_1+L_2}{L_1 + L_2}}$ $k_p = \sqrt{\frac{I_p}{W}}$
NOTE	$W_1 = \frac{L_1}{L_2}W_2 \quad W = W_1+2W_2 = W_2\left(\frac{L_1}{L_2} + 2\right)$	$W_1 = \frac{L_1}{L_2}W_2 \quad W = 2W_1+2W_2 = 2W_2\left(\frac{L_1}{L_2} + 1\right)$

Adapted from Weight Handbook, Vol. I, Society of Aeronautical Weight Engineers, Inc.

	V-ROD	L-ROD
		
LENGTH	$2L$	$L_1 + L_2$
CENTROID	$\bar{x} = \frac{L}{2} \text{SIN } \alpha \qquad \bar{y} = L \text{ COS } \alpha$	$\bar{x} = \frac{L_2^{\,2}}{2(L_1 + L_2)} \qquad \bar{y} = \frac{L_1^{\,2}}{2(L_1 + L_2)}$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{WL^2}{3} \text{COS}^2 \alpha$ $I_{x1} = \frac{4}{3} WL^2 \text{COS}^2 \alpha$ $I_y = \frac{WL^2}{12} \text{SIN}^2 \alpha$ $I_{y_1} = \frac{WL^2}{3} \text{SIN}^2 \alpha$ $I_p = I_x + I_y$ $I_{p_1} = I_{x_1} + I_{y_1}$	$I_x = \frac{W}{12} \; \frac{L_1^3 (L_1 + 4L_2)}{(L_1 + L_2)^2}$ $I_y = \frac{W}{12} \; \frac{L_2^3 (4L_1 + L_2)}{(L_1 + L_2)^2}$ $I_p = I_x + I_y$
RADIUS OF GYRATION	$k_x = 0.577 \text{ L COS } \alpha$ $k_{x1} = 1.15 \text{ L COS } \alpha$ $k_y = 0.289 \text{ L SIN } \alpha$ $k_{y_1} = 0.577 \text{ L SIN } \alpha$ $k_p = 0.289 \text{ L} \sqrt{4 \text{ COS}^2 \alpha + \text{SIN}^2 \alpha}$ $k_{p_1} = 0.577 \text{ L} \sqrt{4 \text{ COS}^2 \alpha + \text{SIN}^2 \alpha}$	$k_x = \frac{0.289 L_1}{L_1 + L_2} \sqrt{L_1 (L_1 + 4L_2)}$ $k_y = \frac{0.289 \, L_2}{L_1 + L_2} \sqrt{L_2 (4L_1 + L_2)}$ $k_p = \sqrt{\frac{I_p}{W}}$
NOTE	....	$W_1 + W_2 = W_2 \; \frac{L_1}{L_2} + 1 \; = W$

Adapted from Weight Handbook, Vol. 1, Society of Aeronautical Weight Engineers, Inc.

	STRAIGHT ROD	INCLINED ROD NOT THROUGH C G AXIS
		
LENGTH	L	$L_1 + L_2$
CENTROID	$\bar{x} = \frac{L}{2}$	
WEIGHT MOMENT OF INERTIA	I_x = NEGLIGIBLE $I_y = I_p = \frac{WL^2}{12}$ $I_{y_1} = I_{p_1} = \frac{WL^2}{3}$ $I_c = \frac{WL^2 \sin^2 \alpha}{12}$ $I_D = \frac{WL^2 \sin^2 \alpha}{3}$	$I_x = \frac{W}{3} \sin^2 \alpha \, (L_1^2 - L_1 L_2 + L_2^2)$ $I_y = \frac{W}{3} \cos^2 \alpha \, (L_1^2 - L_1 L_2 + L_2^2)$ $I_p = I_x + I_y$ NOTE: ASSUME: WT. OF $L_1 = W_1$ WT. OF $L_2 = W_2$ THEN: $W_1 = \frac{L1}{L2} W_2$ $W = W_1 + W_2$ $W_1 + W_2 = W_2 \left(\frac{L1}{L2} + 1\right)$
RADIUS OF GYRATION	k_x = NEGLIGIBLE $k_y = 0.289L$ $k_{y_1} = 0.577L$ $k_c = 0.289L \sin \alpha$ $k_D = 0.577L \sin \alpha$	$k_x = 0.577 \sin \alpha \sqrt{L_1^2 - L_1 L_2 + L_2^2}$ $k_y = 0.577 \cos \alpha \sqrt{L_1^2 - L_1 L_2 + L_2^2}$ $k_p = 0.577 \sqrt{L_1^2 - L_1 L_2 + L_2^2}$

Adapted from Weight Handbook, Vol. I, Society of Aeronautical Weight Engineers, Inc.

PLANE AREAS

SUMMARY OF PLANE AREA PROPERTIES

Centroid by Integration

$$\bar{y} = \frac{\int y\,dA}{A} \qquad \bar{x} = \frac{\int x\,dA}{A}$$

Centroid by Area Moment Summation

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + \cdots + a_n y_n}{a_1 + a_2 + \cdots + a_n}$$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + \cdots + a_n x_n}{a_1 + a_2 + \cdots + a_n}$$

Moments of Inertia

$$I_x = \int y^2 dA$$

$$I_y = \int x^2 dA$$

$$I_z = J_z = \int r^2 dA = I_x + I_y$$

Transfer of Axes

$$I_x = \bar{I}_x + Adx^2$$

$$I_y = \bar{I}_y + Ady^2$$

$$J_z = \bar{J}_z + Ad^2 = I_x + I_y$$

$$k^2 = \bar{k}^2 + d^2$$

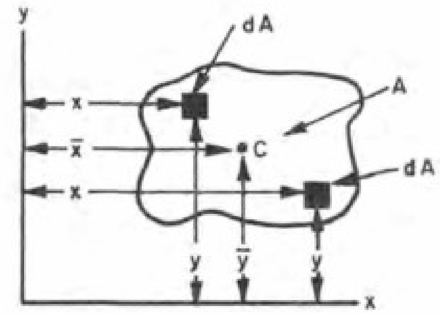
Radius of Gyration

$$k = \sqrt{I/A}$$

CENTROID OF AN AREA

The centroid of an area is represented by a point whose distance from any axis times the total area is equal to the first moment of the area with respect to that axis.

The first moment of an area, often referred to as the statical moment, is the algebraic sum of the moments of the differential parts of the area, with the product of the differential area and the perpendicular distance from the differential area to the axis in question representing the moment of each segment.



Letting Q represent the first moment of the area, the resulting mathematical expressions define the moment of area A

$$Q_x = \int y dA, \qquad Q_y = \int x dA$$

and, for the centroid,

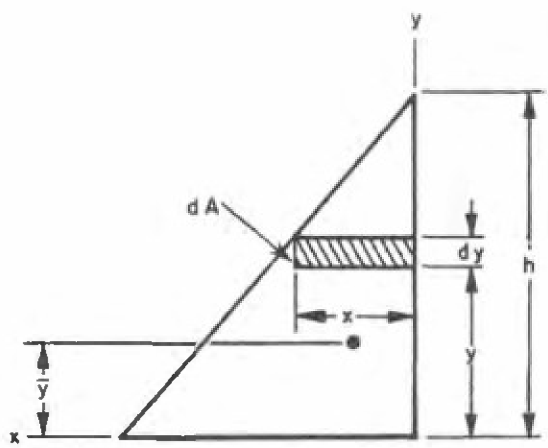
$$A\bar{y} = \int y dA, \qquad A\bar{x} = \int x dA$$

or

$$\bar{y} = \frac{\int y dA}{A}, \qquad \bar{x} = \frac{\int x dA}{A}$$

Examples.

TRIANGLE



$$A = bh/2$$

$$A\bar{y} = \int y dA$$

$$dA = x dy$$

$$A\bar{y} = \int xy dy$$

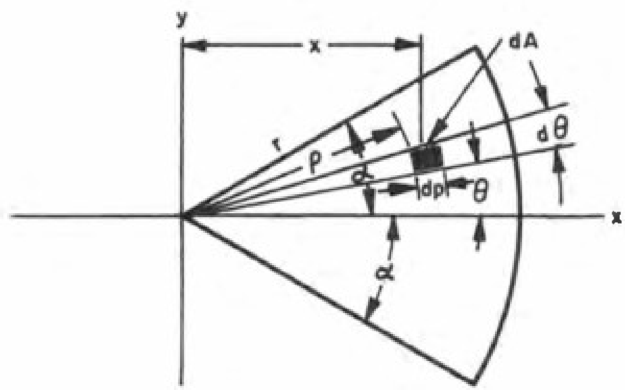
From similar triangles,

$$\frac{x}{h - y} = \frac{b}{h}, \qquad \text{or} \qquad x = \frac{b(h - y)}{h}$$

$$A\bar{y} = \frac{b}{h}\int_0^h (h - y)ydy = \frac{bh^2}{6}$$

$$\bar{y} = \frac{bh^2/6}{bh/2} = \frac{h}{3} \text{ (ans.)}$$

CIRCULAR SECTOR



$$A\bar{x} = \int x dA$$

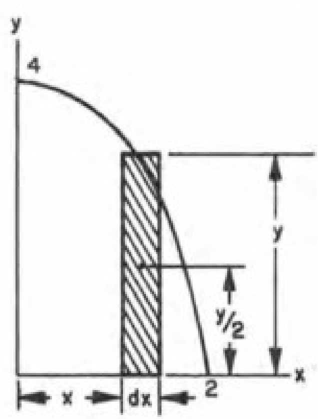
$$x = \rho \cos \theta$$

$$dA = \rho d\theta d\rho$$

$$A\bar{x} = \int_0^r \int_{-\alpha}^{\alpha} \rho \cos \theta \rho d\rho d\theta = \frac{2r^3 \sin \alpha}{3}$$

$$\bar{x} = \frac{(2/3)r^3 \sin \alpha}{A} = \frac{(2/3)r^3 \sin \alpha}{r^2\alpha} = \frac{2r \sin \alpha}{3\alpha} \text{ (ans.)}$$

HALF PARABOLA



Parabola: $y = 4 - x^2$

$dA = ydx$

$$A = \int_0^2 y dx = \int_0^2 (4 - x^2) dx = \frac{16}{3}$$

$$Q_x = \int y dA = \int_0^2 \left(\frac{y}{2}\right)(y) dx = \int_0^2 \frac{y^2}{2} dx = \frac{1}{2} \int_0^2 (4 - x^2)^2 dx = \frac{128}{15}$$

$$Q_y = \int x dA = \int_0^2 xy dx = \int_0^2 (4x - x^3) dx = 4$$

$$\bar{x} = \frac{Q_y}{A} = \frac{4}{16/3} = \frac{3}{4}, \qquad \bar{y} = \frac{Q_x}{A} = \frac{128/15}{16/3} = \frac{8}{5} \quad \text{(ans.)}$$

Note. When summing the elemental strips about the x-axis as shown in the diagram, the moment arm is equal to y/2. If the summation is made with respect to the y-axis, the moment arm is equal to x (not x/2).

CENTROID OF A COMPOSITE AREA

Composite areas have centroids, the coordinates of which may be determined by applying the basic definition to the total area as follows:

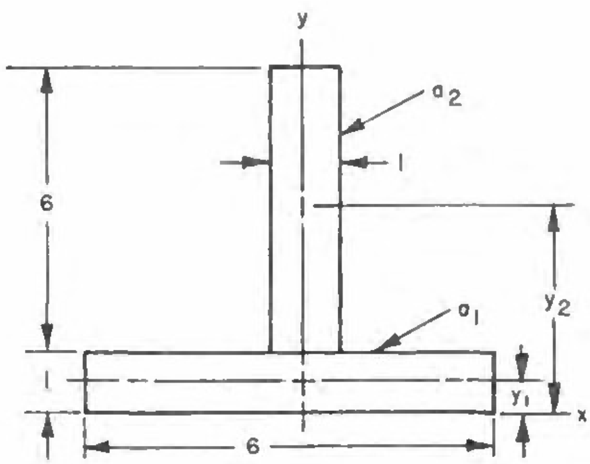
$$A\bar{y} = \Sigma ay \qquad \text{and} \qquad A\bar{x} = \Sigma ax$$

or

$$\bar{y} = \frac{\Sigma ay}{A} \qquad \text{and} \qquad \bar{x} = \frac{\Sigma ax}{A}$$

where x and y are the perpendicular distances from their respective axes to the areas in question, a is the area of each individual part, and A is the total area of the composite body.

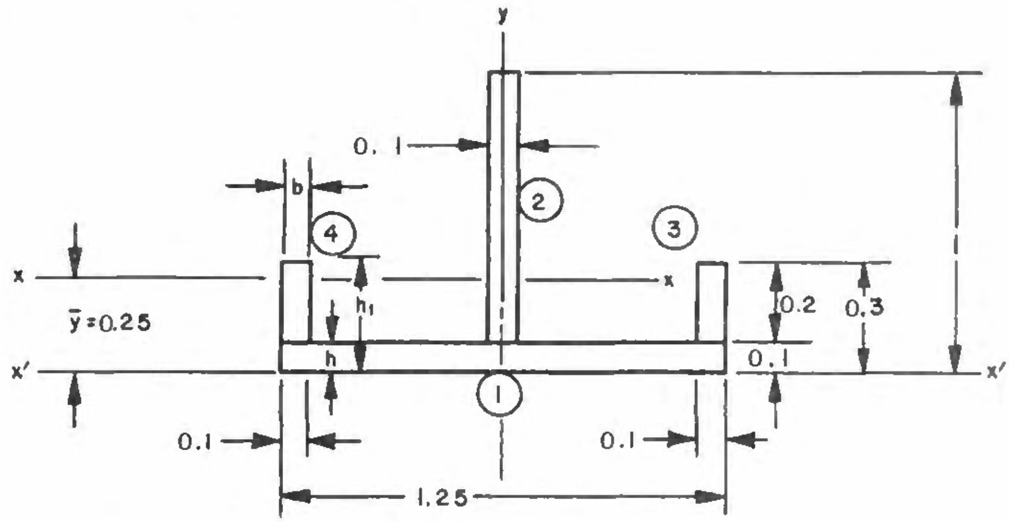
Example.



$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{A} = \frac{(1)(6)(1/2) + (6)(1)(4)}{6 + 6} = 2.25 \text{ in.} \quad \text{(ans.)}$$

$\bar{x} = 0$, by symmetry.

MOMENT OF INERTIA OF A COMPOSITE AREA



$$\bar{y} = \frac{\left[(1.25)(0.1)\right](0.05) + (0.1)(0.9)(0.55) + (2)(0.1)(0.2)(0.2)}{(1.25)(0.1) + (0.1)(0.9) + (2)(0.1)(0.2)}$$

$$= \frac{0.06375}{0.255} = 0.25 \text{ in.} \quad \text{(ans.)}$$

An alternative method for obtaining the moment of inertia of a composite area and the centroidal distance, d, as in the diagram above, is given in Table 1.

TABLE 1. COMPUTATION TABLE FOR THE MOMENT OF INERTIA AND CENTROIDAL DISTANCE OF A COMPOSITE AREA

Part	Area, in^2	b, in.	h_1^2, in^2	h^2, in^2	Moment, $\frac{b(h_1^2 - h^2)}{2}$, in^3	h_1^3, in^3	h^3, in^3	$I_{x'}$ $b/3(h_1^3 - h^3)$, in^4
1	0.125	1.25	0.01	0	0.006	0.001	0	0.00003
2	0.09	0.1	1.0	0.01	0.049	1.000	0.001	0.03333
3	0.02	0.1	0.09	0.01	0.004	0.027	0.001	0.00087
4	0.02	0.1	0.09	0.01	0.004	0.027	0.001	0.00087
A = 0.255			M = 0.06375			$I_{x'}$ = 0.035		

The centroidal distance may be found by either method. By definition,

$$d = \frac{M}{A} = \frac{0.06375}{0.255} = 0.25 = \bar{y}$$

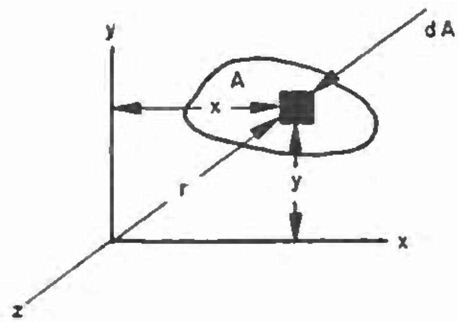
$$I_x = I_{x'} - Ad^2$$

$$= 0.035 - 0.255(0.25)^2$$

$$= 0.0191 \text{ in}^4 \quad \text{(ans.)}$$

MOMENTS OF INERTIA OF A PLANE AREA

The moment of inertia of a plane area, mass, or volume is sometimes referred to as the second moment, since the first moment, Q, is multiplied by the differential area moment arm.



$$I_x = \int y^2 dA$$
$$I_y = \int x^2 dA$$

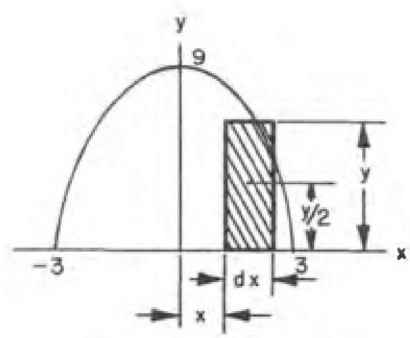
where the elements are integrated over the entire body.

The moment of inertia of the body about the z-axis is

$$I_z = J_z = \int r^2 dA$$

and, since $r^2 = x^2 + y^2$, $J_z = I_x + I_y$ where J_z is known as the polar moment of inertia of the body.

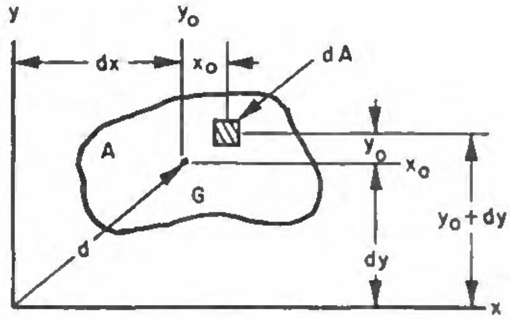
Example.



The moment of inertia of the parabola $y = 9 - x^2$ about the y-axis is calculated as follows

$$dA = ydx$$
$$x^2 = 9 - y$$
$$y = 9 - x^2$$
$$I_y = \int x^2 dA$$
$$= 2\int_0^3 (x^2)(9 - x^2)dx$$
$$= 2\int_0^3 (9x^2 - x^4)dx$$
$$= 2\left[3x^2 + \frac{x^5}{5}\right]_0^3 = \frac{324}{5} \text{ (ans.)}$$

TRANSFER OF AXES ON A PLANE AREA



$$dI_x = (y_0 + dx)^2 dA$$

$$I_x = \int (y_0^2 + 2y_0 dx + dx^2) dA$$

$$= \int y_0^2 dA + 2dx \int y_0 dA + dx^2 \int dA$$

Since the second term in the equation above is zero, the resulting integrations leave

$$I_x = \bar{I}_x + Adx^2$$

and similarly,

$$I_y = \bar{I}_y + Ady^2$$

The sum of these two equations (from $J_z = I_x + I_y$) gives

$$J_z = \bar{J}_z + Ad^2$$

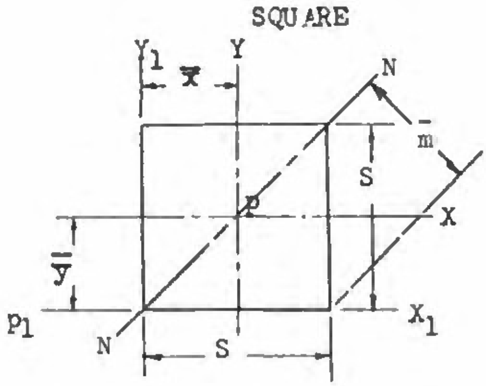
which is the polar moment of inertia of the body when transferred through the distance d.

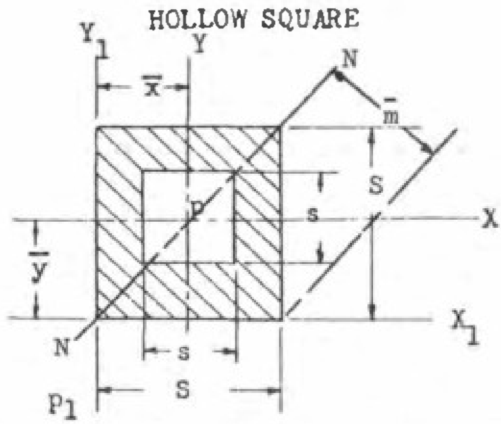
RADIUS OF GYRATION

By definition, $k = \sqrt{I/A}$. Substituting into the moment of inertia equations yields

$$k^2 = \bar{k}^2 + d^2$$

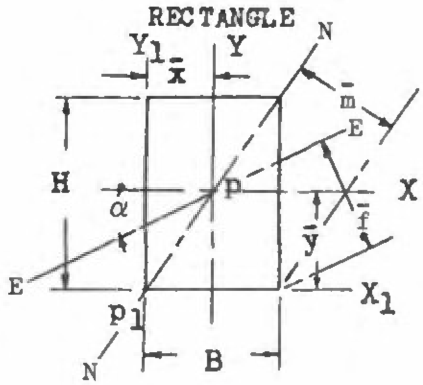
where $\bar{k}$ is the radius of gyration about a centroidal axis parallel to the axis about which k applies, the axes being separated by d.





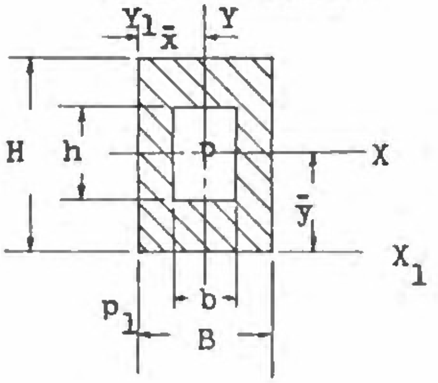
	SQUARE	HOLLOW SQUARE
AREA	S^2	$S^2 - s^2$
CENTROID	$\bar{x} = \bar{y} = \frac{S}{2}$ $\bar{m} = \frac{S}{\sqrt{2}} = 0.7075$	$\bar{x} = \bar{y} = \frac{S}{2}$ $\bar{m} = \frac{S}{\sqrt{2}} = 0.7075$
WEIGHT MOMENT OF INERTIA	$I_x = I_y = \frac{WS^2}{12}$ $I_{x_1} = I_{y_1} = \frac{WS^2}{3}$ $I_p = I_x + I_y = \frac{WS^2}{6}$ $I_{p_1} = I_{x_1} + I_{y_1} = \frac{2WS^2}{3}$	$I_x = I_y = \frac{W(S^2 - s^2)}{12}$ $I_{x1} = I_{y_1} = \frac{W(4S^2 + s^2)}{12} = I_x + \frac{5}{12} WS^2$ $I_p = I_x + I_y = \frac{W(S^2 - s^2)}{6}$ $I_{p_1} = \frac{W(4S^2 + s^2)}{6} = 2 I_{x_1}$
AREA MOMENT OF INERTIA	$I_N = I_x = I_y = \frac{S^4}{12}$ $I_{x_1} = I_{y_1} = \frac{S^4}{3}$ $I_p = I_x + I_y = \frac{S^4}{6}$ $I_{p_1} = I_{x_1} + I_{y_1} = \frac{2S^4}{3}$	$I_N = I_x = I_y = \frac{S^4 - s^4}{12}$ $I_{x_1} = I_{y_1} = \frac{4S^4 - 3S^2s^2 - s^4}{12}$ $I_p = \frac{S^4 - s^4}{6}$ $I_{p_1} = \frac{4S^4 - 3S^2s^2 - s^4}{6}$
RADIUS OF GYRATION	$k_N = k_x = k_y = 0.289S$ $k_{x_1} = k_{y_1} = 0.577S$ $k_p = 0.408S$ $k_{p_1} = 0.816S$	$k_N = k_x = k_y = 0.289 \sqrt{S^2 + s^2}$ $k_{x_1} = \sqrt{\frac{I_{x1}}{S^2 - s^2}} = \frac{1}{6} \sqrt{3(4S^2 + s^2)}$ $k_{y_1} = \sqrt{\frac{I_{y1}}{S^2 - s^2}} = \frac{1}{6} \sqrt{3(4S^2 + s^2}$ $k_p = 0.408 \sqrt{S^2 + s^2}$ $k_{p_1} = \sqrt{\frac{I_{p1}}{S^2 - s^2}} = \frac{1}{6} \sqrt{6(4S^2 + s^2}$

Adapted from Weight Handbook, Vol. 1, Society of Aeronautical Weight Engineers, Inc.



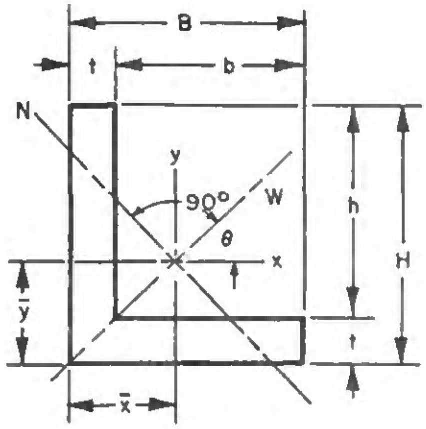
AREA	BH
CENTROID	$\bar{x} = \frac{B}{2}$ $\quad \bar{f} = \frac{B\sin\alpha + H\cos\alpha}{2}$ $\bar{y} = \frac{H}{2}$ $\quad \bar{m} = \frac{BH}{\sqrt{B^2 + H^2}}$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{WH^2}{12}$ $\quad I_y = \frac{WB^2}{12}$ $\quad I_p = \frac{W}{12}(H^2 + B^2)$ $I_{x_1} = \frac{WH^2}{3}$ $\quad I_{y_1} = \frac{WB^2}{3}$ $\quad I_{p_1} = \frac{W}{3}(H^2 + B^2)$
AREA MOMENT OF INERTIA	$I_x = \frac{BH^3}{12}$ $\quad I_y = \frac{HB^3}{12}$ $\quad I_p = \frac{BH}{12}(H^2 + B^2)$ $\quad I_N = \frac{B^3H^3}{6(B^2 + H^2)}$ $I_{x_1} = \frac{BH^3}{3}$ $\quad I_{y_1} = \frac{HB^3}{3}$ $\quad I_{p_1} = \frac{BH}{3}(H^2 + B^2)$ $\quad I_E = \frac{BH(B^2\sin^2\alpha + H^2\cos^2\alpha)}{12}$
RADIUS OF GYRATION	$k_x = 0.289H$ $\quad k_y = 0.289B$ $\quad k_p = 0.289\sqrt{H^2 + B^2}$ $\quad k_N = \frac{BH}{\sqrt{6(B^2 + H^2)}}$ $k_{x_1} = 0.577H$ $\quad k_{y_1} = 0.577B$ $\quad k_{p_1} = 0.577\sqrt{H^2 + B^2}$ $\quad k_E = \sqrt{\frac{B^2\sin^2\alpha + H^2\cos^2\alpha}{12}}$

Adapted from Weight Handbook, Vol. 1, Society of Aeronautical Weight Engineers, Inc.

	HOLLOW RECTANGLE
	
AREA	$BH - bh$
CENTROID	$\bar{x} = \frac{B}{2}$ $\bar{y} = \frac{H}{2}$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{W}{12}\left[\frac{BH^3 - bh^3}{BH - bh}\right]$ $I_y = \frac{W}{12}\left[\frac{HB^3 - hb^3}{BH - bh}\right]$ $I_{x_1} = I_x + \frac{WH^2}{4}$ $I_{y_1} = I_y + \frac{WB^2}{4}$ $I_p = I_x + I_y$ $I_{p_1} = I_{x_1} + I_{y_1}$
AREA MOMENT OF INERTIA	$I_x = \frac{BH^3 - bh^3}{12}$ $I_y = \frac{HB^3 - hb^3}{12}$ $I_{x_1} = \frac{BH^3}{3} - \frac{bh(3H^2 + h^2)}{12}$ $I_{y_1} = I_y + \frac{(BH - bh)B^2}{4}$ $I_p = I_x + I_y$ $I_{p_1} = I_{x_1} + I_{y_1}$
RADIUS OF GYRATION	$k_x = \sqrt{\frac{BH^3 - bh^3}{12(BH-bh)}}$ $k_y = \sqrt{\frac{HB^3 - hb^3}{12(BH-bh)}}$ $k_p = \sqrt{\frac{I_p}{BH - bh}}$ $k_{x_1} = \sqrt{\frac{I_{x_1}}{BH - bh}}$ $k_{y_1} = \sqrt{\frac{I_{y_1}}{BH - bh}}$ $k_{p_1} = \sqrt{\frac{I_{p_1}}{BH - bh}}$

Adapted from Weight Handbook, Vol. I, Society of Aeronautical Weight Engineers, Inc.

ANGLE



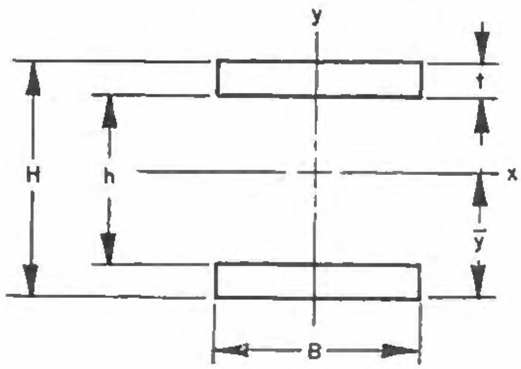
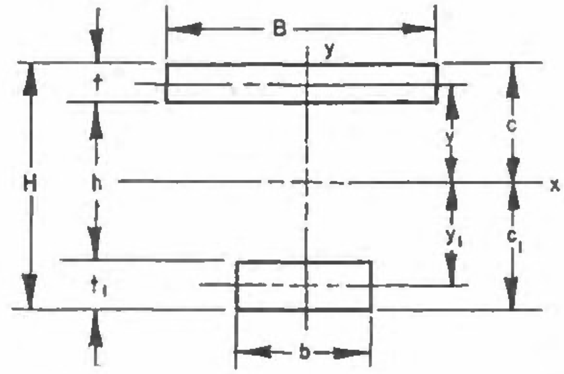
$$\text{Tan } 2\theta = \frac{2I_{xy}}{I_y - I_x}$$

I_{xy} = product of inertia about x-x and y-y

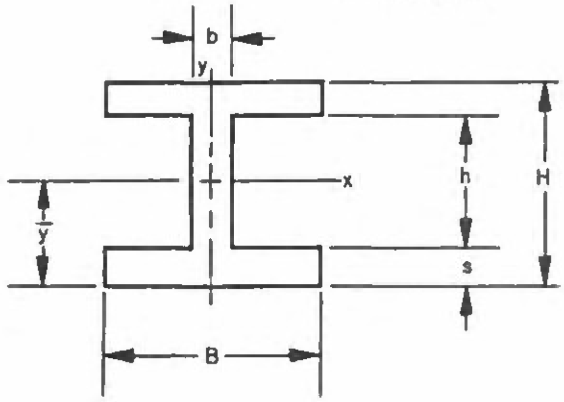
$$I_{xy} = \mp \frac{(bBhHt)}{4(B + h)}$$

I_{xy} is negative when the heel of the angle, with respect to the center of gravity, is in the first or third quadrant; positive when it is in the second or fourth quadrant

AREA	$t(B + h)$
CENTROID	$\bar{x} = \dfrac{B^2 + ht}{2(B + h)}$ $\quad \bar{y} = \dfrac{H^2 + bt}{2(B + h)}$
WEIGHT MOMENT OF INERTIA	$I_x = \dfrac{m}{3}\left[\dfrac{t(H - \bar{y})^3 + B\bar{y}^3 - b(\bar{y} - t)^3}{t(H + B - t)}\right]$ $I_y = \dfrac{m}{3}\left[\dfrac{t(B - \bar{x})^3 + H\bar{x}^3 - h(\bar{x} - t)^3}{t(H + B - t)}\right]$ $I_N = \dfrac{m(I_x \sin^2 \theta + I_y \cos^2 \theta + I_{xy} \sin 2\theta)}{t(H + B - t)}$ $I_W = \dfrac{m(I_x \cos^2 \theta + I_y \sin^2 \theta - I_{xy} \sin 2\theta)}{t(H + B - t)}$
AREA MOMENT OF INERTIA	$I_x = 1/3\left[t(H - \bar{y})^3 + B\bar{y}^3 - b(\bar{y} - t)^3\right]$ $I_y = 1/3\left[t(B - \bar{x})^3 + H\bar{x}^3 - h(\bar{x} - t)^3\right]$ $I_N = I_x \sin^2 \theta + I_y \cos^2 \theta + I_{xy} \sin 2\theta$ $I_W = I_x \cos^2 \theta + I_y \sin^2 \theta - I_{xy} \sin 2\theta$
RADIUS OF GYRATION	$k = \sqrt{I/A}$

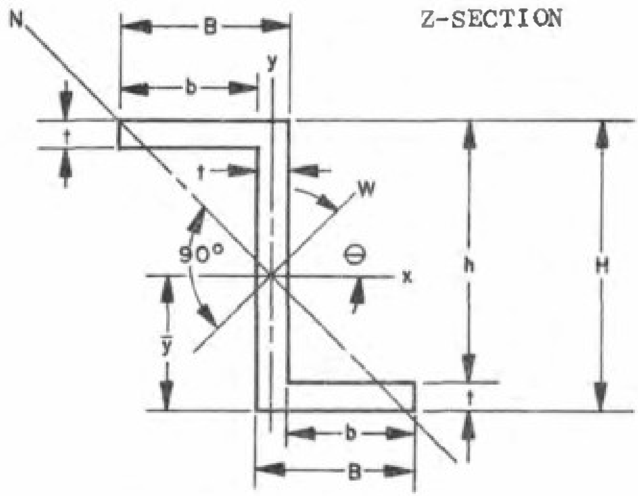
	EQUAL RECTANGLES	UNEQUAL RECTANGLES
		
AREA	$B(H - h)$	$Bt + bt_1 \qquad c_1 = H - c$
CENTROID	$\bar{y} = H/2$	$c = \dfrac{(1/2)Bt^2 + bt_1\left[H - (1/2)t_1\right]}{Bt + bt_1}$
WEIGHT MOMENT OF INERTIA	$I_x = \dfrac{m(H^3 - h^3)}{12(H - h)}$ $I_y = \dfrac{mB^2}{12}$	$I_x = \dfrac{m(Bt^3 + 12Bty^2 + bt_1^3 + 12bt_1y_1^2)}{12(Bt + bt_1)}$ $I_y = \dfrac{m(tB^3 + t_1b^3)}{12(Bt + bt_1)}$
AREA MOMENT OF INERTIA	$I_x = \dfrac{B(H^3 - h^3)}{12}$ $I_y = \dfrac{B^3(H - h)}{12}$	$I_x = \dfrac{Bt^3}{12} + Bty^2 + \dfrac{bt_1^3}{12} + bt_1y_1^2$ $I_y = \dfrac{tB^3 + t_1b^3}{12}$
RADIUS OF GYRATION	$k_x = \sqrt{\dfrac{H^3 - h^3}{12B(H - h)}}$ $k_y = \sqrt{\dfrac{B^3(H - h)}{12B(H - h)}} = 0.289B$	$k_x = \sqrt{\dfrac{Bt^3 + 12Bty^2 + bt_1^3 + 12bt_1y_1^2}{12(Bt + bt_1)}}$ $k_y = \sqrt{\dfrac{tB^3 + t_1b^3}{12(Bt + bt_1)}}$

H-SECTION

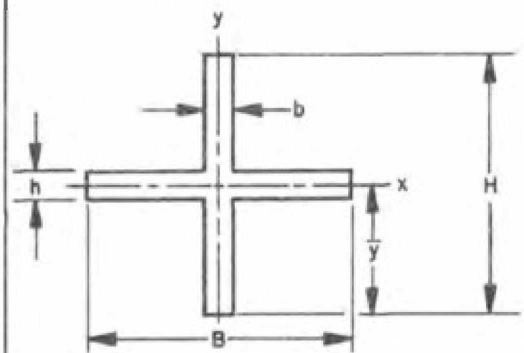


AREA	$BH - h(B - b)$
CENTROID	$\bar{y} = H/2 \qquad \bar{x} = 0$
WEIGHT MOMENT OF INERTIA	$I_x = \dfrac{m\left[BH^3 - h^3(B - b)\right]}{12\left[BH - h(B - b)\right]}$ $I_y = \dfrac{m(hb^3 + 2sB^3)}{12\left[BH - h(B - b)\right]}$
AREA MOMENT OF INERTIA	$I_x = \dfrac{BH^3 - h^3(B - b)}{12}$ $I_y = \dfrac{hb^3 + 2sB^3}{12}$
RADIUS OF GYRATION	$k_x = \sqrt{\dfrac{BH^3 - h^3(B - b)}{12\left[BH - h(B - b)\right]}}$ $k_y = \sqrt{\dfrac{hB^3 + 2sB^3}{12\left[BH - h(B - b)\right]}}$

Z-SECTION

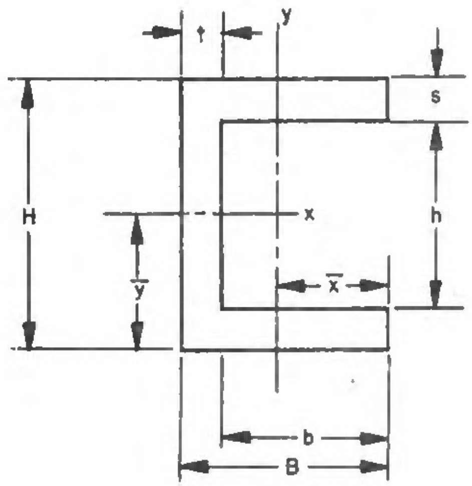


CROSSED RECTANGLES



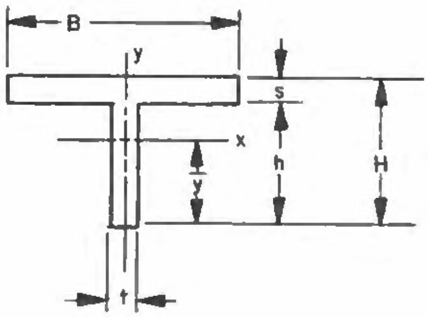
	Z-SECTION	CROSSED RECTANGLES
AREA	$t(H + 2b)$	$Hb + h(B - b)$
CENTROID	$\bar{y} = H/2 \quad \bar{x} = 0$	$\bar{y} = H/2 \quad \bar{x} = B/2$
WEIGHT MOMENT OF INERTIA	$\tan 2\theta = \dfrac{(Ht - t^2)(B^2 - Bt)}{I_x - I_y}$ $I_x = \dfrac{m\left[BH^3 - b(H - 2t)^3\right]}{12\left[t(H + 2b)\right]}$ $I_y = \dfrac{m\left[H(B + b)^3 - 2b^3h - 6bB^2h\right]}{12t(H + 2b)}$	$I_x = \dfrac{m\left[bH^3 + h^3(B - b)\right]}{12\left[Hb + h(B - b)\right]}$ $I_y = \dfrac{m\left[hB^3 + b^3(H - h)\right]}{12\left[Hb + h(B - b)\right]}$
AREA MOMENT OF INERTIA	$I_x = \dfrac{BH^3 - b(H - 2t)^3}{12}$ $I_y = \dfrac{H(B + b)^3 - 2b^3h - 6B^2bh}{12}$ $I_N = I_x \sin^2 \theta + I_y \cos^2 \theta + I_{xy} \sin 2\theta$ $I_W = I_x \cos^2 \theta + I_y \sin^2 \theta - I_{xy} \sin 2\theta$	$I_x = \dfrac{bH^3 + h^3(B - b)}{12}$ $I_y = \dfrac{hB^3 + b^3(H - h)}{12}$
RADIUS OF GYRATION	$k_x = \sqrt{I_x/A}$ $k_y = \sqrt{I_y/A}$	$k_x = \sqrt{\dfrac{bH^3 + h^3(B - b)}{12\left[Hb + h(B - b)\right]}}$ $k_y = \sqrt{\dfrac{hB^3 + b^3(H - h)}{12\left[Hb + h(B - b)\right]}}$

CHANNEL OR U-SECTION



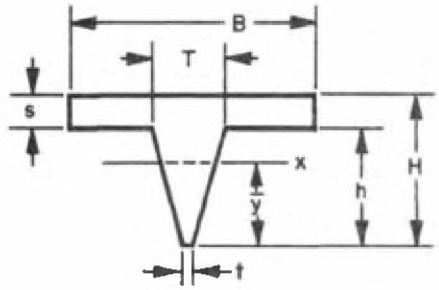
AREA	$BH - h(B - t) = A$
CENTROID	$\bar{y} = \frac{H}{2}$ $\quad \bar{x} = B - \frac{2B^2 s + ht^2}{2BH - 2h(B - t)}$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{m\left[BH^3 - h^3(B - t)\right]}{12\left[BH - h(B - t)\right]}$ $I_y = \frac{m}{3}\left[\frac{2sB^3 + ht^3 - 3A(B - \bar{x})^2}{BH - h(B - t)}\right]$
AREA MOMENT OF INERTIA	$I_x = \frac{BH^3 - h^3(B - t)}{12}$ $I_y = \frac{2sB^3 + ht^3}{3} - A(B - \bar{x})^2$
RADIUS OF GYRATION	$k_x = \sqrt{\frac{BH^3 - h^3(B - t)}{12\left[BH - h(B - t)\right]}}$ $k_y = \sqrt{\frac{2sB^3 + ht^3 - 3A(B - \bar{x})^2}{3\left[BH - h(B - t)\right]}}$

T-SECTION



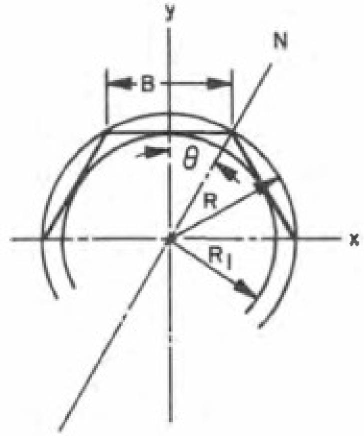
AREA	$Bs + ht$
CENTROID	$\bar{y} = H - \dfrac{H^2 t + s^2(B - t)}{2(Bs + ht)}$
WEIGHT MOMENT OF INERTIA	$I_x = \dfrac{m}{3}\left[\dfrac{t\bar{y}^3 + B(H - \bar{y})^3 - (B - t)(H - \bar{y} - s)^3}{Bs + ht}\right]$ $I_y = \dfrac{m}{12}\left(\dfrac{sB^3 + ht^3}{Bs + ht}\right)$
AREA MOMENT OF INERTIA	$I_x = \dfrac{t\bar{y}^3 + B(H - \bar{y})^3 - (B - t)(H - \bar{y} - s)^3}{3}$ $I_y = \dfrac{sB^3 + ht^3}{12}$
RADIUS OF GYRATION	$k_x = \sqrt{I_x/A}$ $k_y = \sqrt{I_y/A}$

MODIFIED T-SECTION



AREA	$Bs + \frac{h(T + t)}{2} = A$
CENTROID	$\bar{y} = H - \frac{\left[3Bs^2 + 3ht(H + s) + h(T + t)(h + 3s)\right]}{6A}$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{m\left\{\left[4Bs^3 + h^3(3t + T)\right] - 12A(H - \bar{y} - s)^2\right\}}{6\left[2Bs + h(T + t)\right]}$
AREA MOMENT OF INERTIA	$I_x = \frac{4Bs^3 + h^3(3t + T)}{12} - A(H - \bar{y} - s)^2$
RADIUS OF GYRATION	$k_x = \sqrt{I_x/A}$

REGULAR POLYGON

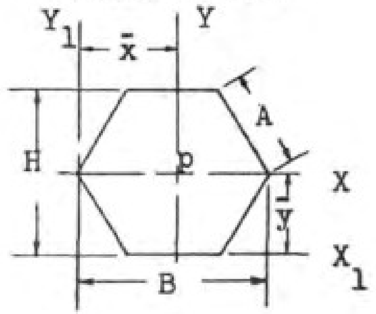


n = number of sides

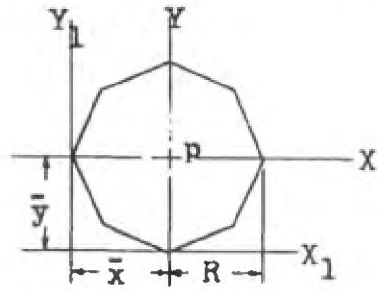
$$\theta = \frac{180°}{n} \qquad B = 2\sqrt{R^2 - r_1^2}$$

AREA	$\frac{nB^2 \cot\theta}{4} = \frac{nR^2 \sin 2\theta}{2} = nR_1^2 \tan\theta$
CENTROID	$\bar{x} = \bar{y} = 0$
WEIGHT MOMENT OF INERTIA	$R = \frac{B}{2\sin\theta} \qquad R_1 = \frac{B}{2\tan\theta}$ $I_y = I_N = \frac{m(6R^2 - B^2)}{24} = \frac{m(12R_1^2 + B^2)}{48}$
AREA MOMENT OF INERTIA	$I_y = I_N = \frac{A(6R^2 - B^2)}{24} = \frac{A(12R_1^2 + B^2)}{48}$
RADIUS OF GYRATION	$k_y = k_N = \sqrt{\frac{6R^2 - B^2}{24}} = \sqrt{\frac{12R_1^2 + B^2}{48}}$

REGULAR HEXAGON



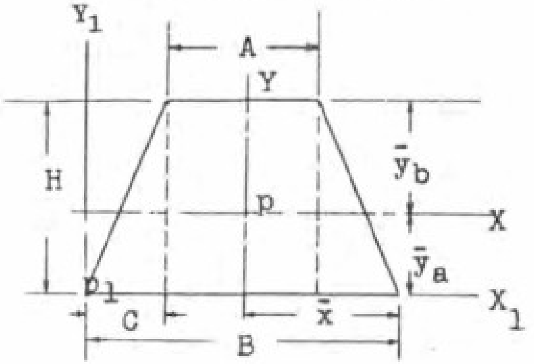
REGULAR OCTAGON



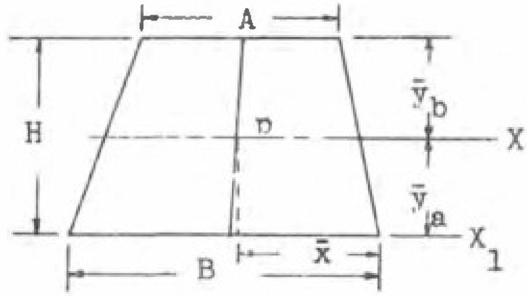
	REGULAR HEXAGON	REGULAR OCTAGON
AREA	$0.866\ H^2$	$2.8284\ R^2$
CENTROID	$\bar{x} = \frac{B}{2} = A \qquad \bar{y} = \frac{H}{2}$	$\bar{x} = \bar{y} = R$
WEIGHT MOMENT OF INERTIA	$I_x = I_y = 0.0694WH^2 = 0.0521WB^2$ $I_{x_1} = 0.3194WH^2$ $I_p = 0.1389WH^2 = 0.1042WB^2$ $I_{y_1} = 0.4028WH^2 = 0.3021WB^2$	$I_x = I_y = 0.2256WR^2$ $I_{x_1} = I_{y_1} = 1.2256WR^2$ $I_p = 0.4512WR^2$
AREA MOMENT OF INERTIA	$I_x = I_y = 0.0601H^4$ $I_{x_1} = 0.2766H^4$ $I_{y_1} = 0.3488H^4$ $I_p = 0.1203H^4$	$I_x = I_y = 0.6381R^4$ $I_{x_1} = I_{y_1} = 3.4665R^4$ $I_p = 1.2761R^4$
RADIUS OF GYRATION	$k_x = k_y = 0.2635H = 0.2282B$ $k_{x_1} = 0.5652H$ $k_{y_1} = 0.6346H$ $k_p = 0.3727H$	$k_x = k_y = 0.4750R$ $k_{x_1} = k_{y_1} = 1.1071R$ $k_p = 0.672R$

Adapted from Weight Handbook, Vol. 1, Society of Aeronautical Weight Engineers, Inc.

ISOSCELES TRAPEZOID

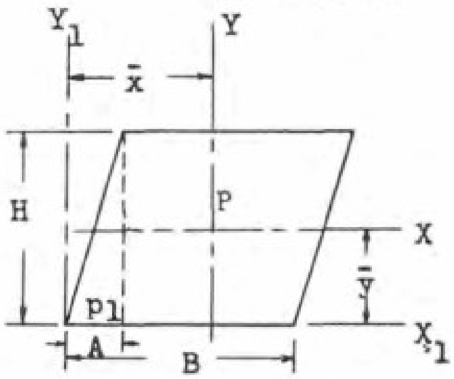
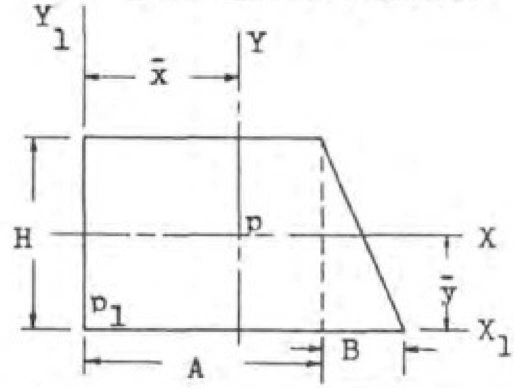


OBLIQUE TRAPEZOID



AREA	$\frac{H(A+B)}{2}$	$\frac{H(A+B)}{2}$
CENTROID	$\bar{x} = \frac{B}{2}$ $\bar{y}_a = \frac{H(B+2A)}{3(B+A)}$ $\bar{y}_b = \frac{H(A+2B)}{3(A+B)}$	$\bar{x}$ is on a line connecting mid-points of sides A & B. $\bar{y}_a = \frac{H(B+2A)}{3(A+B)}$ $\bar{y}_b = \frac{H(2B+A)}{3(A+B)}$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{WH^2}{18}\left(1 + \frac{2AB}{(A+B)^2}\right)$ $I_{x_1} = \frac{WH^2(2A+C)}{6(A+C)}$ or $\frac{WH^2(3A+B)}{6(A+B)}$ $I_y = \frac{W}{24}(A^2+B^2)$ $I_{y_1} = \frac{W}{24}(A^2+7B^2)$ $I_p = I_x + I_y$ $I_{p_1} = I_{x_1} + I_{y_1}$	$I_x = \frac{WH^2}{18}\left(1 + \frac{2AB}{(A+B)^2}\right)$ $I_{x_1} = \frac{WH^2}{6}\left(\frac{3A+B}{A+B}\right)$
AREA MOMENT OF INERTIA	$I_x = \frac{H^3(A^2+4AB+B^2)}{36(A+B)}$ $I_{x1} = \frac{H^3(A+B)(2A+C)}{12(A+C)}$ $I_y = \frac{H(A+B)(A^2+B^2)}{48}$ $I_{y1} = \frac{H(A+B)(A^2+7B^2)}{48}$ $I_p = I_x + I_y$ $I_{p1} = I_{x_1} + I_{y_1}$	$I_x = \frac{H^3(A^2+4AB+B^2)}{36(A+B)}$ $I_{x_1} = \frac{H^3(B+3A)}{12}$
RADIUS OF GYRATION	$k_x = H\sqrt{\frac{2(B^2+4AB+A^2)}{6(A+B)}}$ $k_{x_1} = \sqrt{\frac{H^2(2A+C)}{A+C}}$ $k_y = \sqrt{\frac{A^2+B^2}{24}}$ $k_{y_1} = \sqrt{\frac{A^2+7B^2}{24}}$ $k_p = \sqrt{\frac{2I_p}{H(A+B)}}$ $k_{p_1} = \sqrt{\frac{2I_{p1}}{H(A+B)}}$	$k_x = H\sqrt{\frac{2(A^2+4AB+B^2)}{6(A+B)}}$ $k_{x_1} = H\sqrt{\frac{3A+B}{6(A+B)}}$

Adapted from Weight Handbook, Vol. 1, Society of Aeronautical Weight Engineers, Inc.

	PARALLELOGRAM	RIGHT-ANGLED TRAPEZOID
		
AREA	BH	$\frac{H}{2}(2A + B)$
CENTROID	$\bar{x} = \frac{A + B}{2}$ $\bar{y} = \frac{H}{2}$	$\bar{x} = \frac{3A^2 + 3AB + B^2}{3(2A + B)}$ $\bar{y} = \frac{H(3A + B)}{3(2A + B)}$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{WH^2}{12}$ $I_{x_1} = \frac{WH^2}{3}$ $I_y = \frac{W}{12}(A^2 + B^2)$ $I_{y_1} = \frac{W}{6}(2A^2 + 2B^2 + 3AB)$ $I_p = \frac{W}{12}(A^2 + B^2 + H^2)$ $I_{p_1} = \frac{W(2A^2 + 2B^2 + 3AB + 2H^2)}{6}$	$I_x = \frac{WH^2(6A^2 + 6AB + B^2)}{18(2A + B)^2}$ $I_{x_1} = \frac{WH^2(4A + B)}{6(2A + B)}$ $I_y = I_{y_1} - W\bar{x}^2$ $I_{y_1} = W\left[\frac{6A^2(2A+3B) + B^2(12A+5B)}{18(2A+B)}\right]$ $I_p = I_x + I_y$ $I_{p_1} = I_{x_1} + I_{y_1}$
AREA MOMENT OF INERTIA	$I_x = \frac{BH^3}{12}$ $I_{x_1} = \frac{BH^3}{3}$ $I_y = \frac{BH(A^2 + B^2)}{12}$ $I_{y_1} = \frac{BH}{6}(2A^2+2B^2+3AB)$ $I_p = \frac{BH}{12}(A^2 + B^2 + H^2)$ $I_{p_1} = \frac{BH}{6}(2A^2+2B^2+3AB+2H^2)$	$I_x = \frac{H^3(6A^2+6AB+B^2)}{36(2A+B)}$ $I_{x_1} = \frac{H^3(4A+B)}{12}$ $I_y = I_{y_1} - \frac{H(3A^2+3AB+B^2)^2}{18(2A+B)}$ $I_{y_1} = \frac{H[6A^2(2A+3B) + B^2(12A+5B)]}{36}$ $I_p = I_x + I_y$ $I_{p_1} = I_{x_1} + I_{y_1}$
RADIUS OF GYRATION	$k_x = 0.289H$ $k_{x_1} = 0.577H$ $k_y = 0.289\sqrt{A^2 + B^2}$ $k_{y_1} = 0.408\sqrt{2A^2+2B^2+3AB}$ $k_p = 0.289\sqrt{A^2+B^2+H^2}$ $k_{p_1} = 0.408\sqrt{2A^2+2B^2+3AB+2H^2}$	$k_x = \frac{0.236H}{(2A+B)}\sqrt{6A^2 + 6AB + B^2}$ $k_{x_1} = 0.408H\sqrt{\frac{4A + B}{2A + B}}$ $k_y = \sqrt{\frac{2I_y}{H(2A + B)}}$ $k_{y_1} = \sqrt{\frac{6A^2(2A+3B) + B^2(12A+5B)}{18(2A+B)}}$ $k_p = \sqrt{\frac{2I_p}{H(2A + B)}}$ $k_{p_1} = \sqrt{\frac{2I_{p_1}}{H(2A+B)}}$

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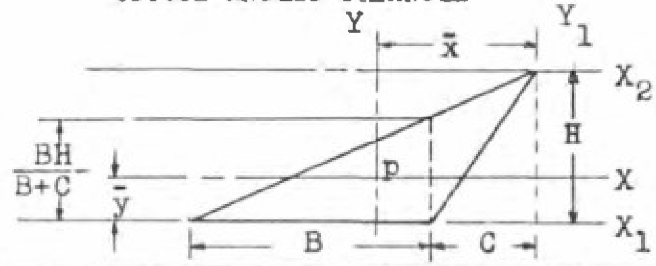
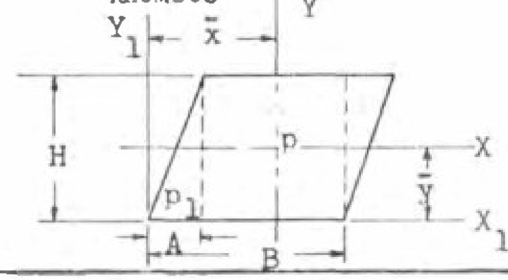
	OBTUSE-ANGLED TRIANGLE	RHOMBUS
		
AREA	$\frac{BH}{2}$	BH
CENTROID	$\bar{x} = \frac{B + 2C}{3}$ $\bar{y} = \frac{H}{3}$	$\bar{x} = \frac{A+B}{2}$ $\bar{y} = \frac{H}{2}$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{WH^2}{18}$ $I_{x_1} = \frac{WH^2}{6}$ $I_{x_2} = \frac{WH^2}{2}$ $I_y = \frac{W}{18}(B^2+BC+C^2)$ $I_{y_1} = \frac{W(B^2+3BC+3C^2)}{6}$ $I_p = \frac{W(H^2+B^2+BC+C^2)}{18}$	$I_x = \frac{WH^2}{12}$ $I_{x_1} = \frac{WH^2}{3}$ $I_y = \frac{W(A^2+B^2)}{12}$ $I_{y_1} = \frac{W(2A^2+2B^2+3AB)}{6}$ $I_p = \frac{WB^2}{6}$ $I_{p_1} = \frac{WB(3A+4B)}{6}$
AREA MOMENT OF INERTIA	$I_x = \frac{BH^3}{36}$ $I_{x_1} = \frac{BH^3}{12}$ $I_{x_2} = \frac{BH^3}{4}$ $I_y = \frac{BH}{36}(B^2 + BC + C^2)$ $I_{y_1} = \frac{BH}{12}(B^2 + 3BC + 3C^2)$ $I_p = \frac{BH}{36}(H^2 + B^2 + BC + C^2)$	$I_x = \frac{BH^3}{12}$ $I_{x_1} = \frac{BH^3}{3}$ $I_y = \frac{BH(A^2 + B^2)}{12}$ $I_{y_1} = \frac{BH(2A^2 + 2B^2 + 3AB)}{6}$ $I_p = \frac{1}{6}B^3H$ $I_{p_1} = \frac{B^2H(3A + 4B)}{6}$
RADIUS OF GYRATION	$k_x = 0.236H$ $k_{x_1} = 0.408H$ $k_{x_2} = 0.707H$ $k_y = 0.236\sqrt{B^2 + BC + C^2}$ $k_{y_1} = 0.408\sqrt{B^2 + 3BC + C^2}$ $k_p = 0.236\sqrt{H^2 + B^2 + BC + C^2}$	$k_x = 0.289H$ $k_{x_1} = 0.577H$ $k_y = 0.289\sqrt{(A^2 + B^2)}$ $k_{y_1} = 0.408\sqrt{2A^2 + 2B^2 + 3AB}$ $k_p = 0.408B$ $k_{p_1} = 0.408\sqrt{B(3A + 4B)}$

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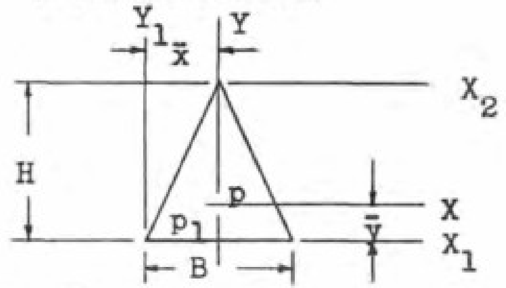
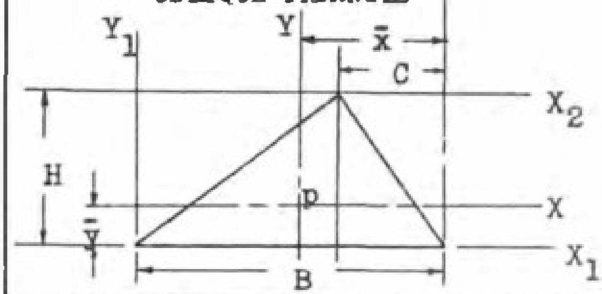
	ISOSCELES TRIANGLE	OBLIQUE TRIANGLE
		
AREA	$\frac{BH}{2}$	$\frac{BH}{2}$
CENTROID	$\bar{x} = \frac{B}{2}$ $\bar{y} = \frac{H}{3}$	$\bar{y} = \frac{H}{3}$ $\bar{x} = \frac{B+C}{3}$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{WH^2}{18}$ $I_{x1} = \frac{WH^2}{6}$ $I_{x2} = \frac{WH^2}{2}$ $I_y = \frac{WB^2}{24}$ $I_{y1} = \frac{7WB^2}{24}$ $I_p = \frac{W(4H^2 + 3B^2)}{72}$ $I_{p1} = \frac{W(7B^2 + 4H^2)}{24}$	$I_x = \frac{WH^2}{18}$ $I_{x1} = \frac{WH^2}{6}$ $I_{x2} = \frac{WH^2}{2}$ $I_y = \frac{W}{18}(B^2 + C^2 - BC)$ $I_p = \frac{W}{18}(H^2 + B^2 + C^2 - BC)$
AREA MOMENT OF INERTIA	$I_x = \frac{BH^3}{36}$ $I_{x1} = \frac{BH^3}{12}$ $I_{x2} = \frac{BH^3}{4}$ $I_y = \frac{B^3H}{48}$ $I_{y1} = \frac{7B^3H}{48}$ $I_p = \frac{4BH^3 + 3B^3H}{144}$ $I_{p1} = \frac{4BH^3 + 7B^3H}{48}$	$I_x = \frac{BH^3}{36}$ $I_{x1} = \frac{BH^3}{12}$ $I_{x2} = \frac{BH^3}{4}$ $I_y = \frac{BH}{36}(B^2 + C^2 - BC)$ $I_p = \frac{BH}{36}(H^2 + B^2 + C^2 - BC)$
RADIUS OF GYRATION	$k_x = 0.236H$ $k_{x1} = 0.408H$ $k_{x2} = 0.707H$ $k_y = 0.204B$ $k_{y1} = 0.540B$ $k_p = 0.118\sqrt{4H^2 + 3B^2}$ $k_{p1} = 0.204\sqrt{4H^2 + 7B^2}$	$k_x = 0.236H$ $k_{x1} = 0.408H$ $k_{x2} = 0.707H$ $k_y = 0.236\sqrt{B^2 + C^2 - BC}$ $k_p = 0.236\sqrt{H^2 + B^2 + C^2 - BC}$

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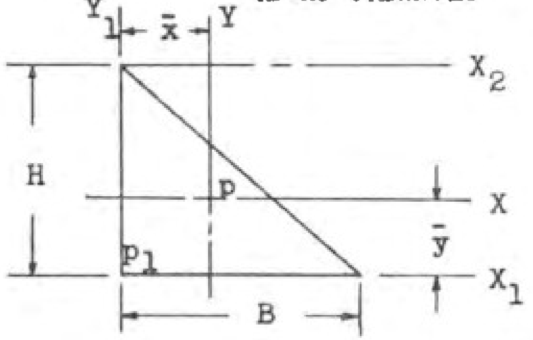
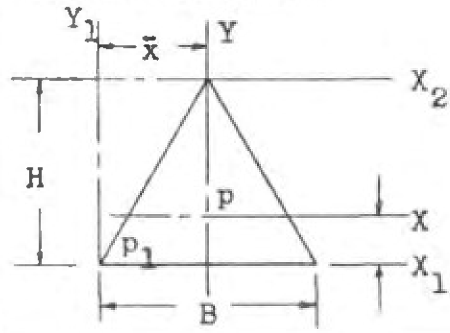
	RIGHT TRIANGLE	EQUILATERAL TRIANGLE
		
AREA	$\frac{BH}{2}$	$\frac{BH}{2}$
CENTROID	$\bar{x} = \frac{B}{3}$ $\bar{y} = \frac{H}{3}$	$\bar{x} = \frac{B}{2}$ $\bar{y} = \frac{H}{3}$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{WH^2}{18}$ $I_{x_1} = \frac{WH^2}{6}$ $I_{x_2} = \frac{WH^2}{2}$ $I_y = \frac{WB^2}{18}$ $I_{y_1} = \frac{WB^2}{6}$ $I_p = \frac{W(H^2+B^2)}{18}$ $I_{p_1} = \frac{W(H^2+B^2)}{6}$	$I_x = I_y = \frac{WB^2}{24}$ $I_{x_1} = \frac{WB^2}{8}$ $I_{x_2} = \frac{WH^2}{2}$ $I_{y_1} = \frac{7WB^2}{24}$ $I_p = \frac{WB^2}{12}$ $I_{p_1} = \frac{5WB^2}{12}$
AREA MOMENT OF INERTIA	$I_x = \frac{BH^3}{36}$ $I_{x_1} = \frac{BH^3}{12}$ $I_{x_2} = \frac{BH^3}{4}$ $I_y = \frac{B^3H}{36}$ $I_{y_1} = \frac{B^3H}{12}$ $I_p = \frac{BH(B^2+H^2)}{36}$ $I_{p_1} = \frac{BH(B^2+H^2)}{12}$	$I_x = I_y = \frac{B^3H}{48}$ $I_{x_1} = \frac{B^3H}{16}$ $I_{x_2} = \frac{BH^3}{4}$ $I_{y_1} = \frac{7B^3H}{48}$ $I_p = \frac{B^3H}{24}$ $I_{p_1} = \frac{5B^3H}{24}$
RADIUS OF GYRATION	$k_x = 0.236H$ $k_{x_1} = 0.408H$ $k_{x_2} = 0.707H$ $k_y = 0.236B$ $k_{y_1} = 0.408B$ $k_p = 0.236\sqrt{B^2+H^2}$ $k_{p_1} = 0.408\sqrt{B^2+H^2}$	$k_x = 0.204B$ $k_{x_1} = 0.354B$ $k_y = 0.204B$ $k_{x_2} = 0.707H$ $k_{y_1} = 0.540B$ $k_p = 0.289B$ $k_{p_1} = 0.456B$

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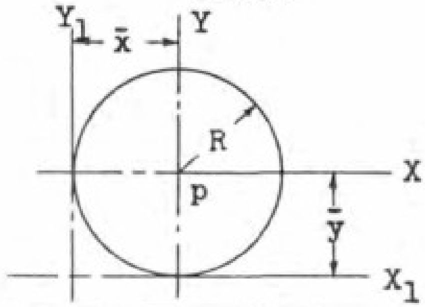
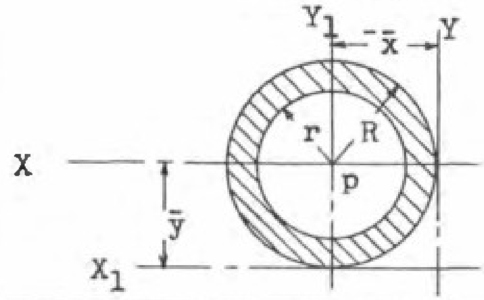
	CIRCLE	HOLLOW CIRCLE
	 	
AREA	$0.7854\ D^2$	$\pi\ (R^2 - r^2)$
CENTROID	$\bar{x} = \bar{y} = R$	$\bar{x} = \bar{y} = R$
WEIGHT MOMENT OF INERTIA	$I_x = I_y = \frac{WD^2}{16} = \frac{WR^2}{4}$ $I_{x_1} = I_{y_1} = 1.25\ WR^2$ $I_p = \frac{WD^2}{8} = \frac{WR^2}{2}$	$I_x = I_y = \frac{W(R^2 + r^2)}{4}$ $I_p = \frac{W(R^2 + r^2)}{2}$ $I_{x_1} = I_{y_1} = \frac{W(5R^2 + r^2)}{4}$
AREA MOMENT OF INERTIA	$I_x = I_y = 0.0491D^4$ $I_{x_1} = I_{y_1} = 0.2454D^4$ $I_p = 0.0982D^4$	$I_x = I_y = \frac{\pi(R^4 - r^4)}{4}$ $I_p = \frac{\pi(R^4 - r^4)}{2}$ $I_{x_1} = I_{y_1} = \frac{\pi(5R^4 - 4R^2r^2 - r^4)}{4}$
RADIUS OF GYRATION	$k_x = k_y = \frac{D}{4}$ $k_{x_1} = k_{y_1} = 0.5590D = 1.118R$ $k_p = 0.3536D$	$k_x = k_y = \frac{1}{2}\sqrt{R^2 + r^2}$ $k_p = \sqrt{\frac{R^2 + r^2}{2}}$ $k_{x_1} = k_{y_1} = \frac{1}{2}\sqrt{5R^2 + r^2}$

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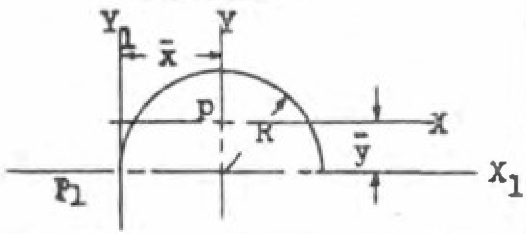
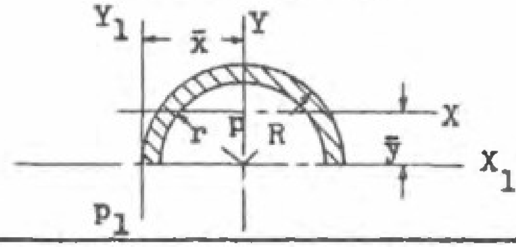
	SEMICIRCLE	HOLLOW SEMICIRCLE
		
AREA	$0.3927\ D^2 = 1.571R^2$	$\frac{\pi(R^2 - r^2)}{2}$
CENTROID	$\bar{x} = R \quad \bar{y} = 0.2122D = 0.4244R$	$\bar{x} = R \quad \bar{y} = 0.4244\left(R + \frac{r^2}{R + r}\right)$
WEIGHT MOMENT OF INERTIA	$I_x = 0.06987\ WR^2 = 0.01747\ WD^2$ $I_{x_1} = 0.25\ WR^2 = 0.0625\ WD^2$ $I_y = 0.25\ WR^2 = 0.0625\ WD^2$ $I_{y_1} = 1.25\ WR^2 = 0.3125\ WD^2$ $I_p = 0.3199\ WR^2 = 0.0800\ WD^2$ $I_{p_1} = 1.50\ WR^2 = 0.3750\ WD^2$	$I_x = \frac{W(R^2 + r^2)}{4} - W\bar{y}^2 \quad I_y = \frac{W(R^2+r^2)}{4}$ $I_p = I_x + I_y$ $I_{x1} = \frac{W(R^2 + r^2)}{4} \quad I_{y1} = \frac{W(5R^2 + r^2)}{4}$ $I_{p1} = \frac{W(3R^2 + r^2)}{2}$
AREA MOMENT OF INERTIA	$I_x = 0.1098\ R^4 = 0.00686\ D^4$ $I_{x_1} = 0.3927\ R^4 = 0.02454\ D^4$ $I_y = 0.3927\ R^4 = 0.02454\ D^4$ $I_{y_1} = 1.9635\ R^4 = 0.12272\ D^4$ $I_p = 0.5025\ R^4 = 0.03140\ D^4$ $I_{p_1} = 2.3562\ R^4 = 0.14726\ D^4$	$I_x = \frac{\pi}{8}(R^4-r^4) - \frac{\pi(R^2-r^2)}{2}\bar{y}^2$ $I_{x1} = \frac{\pi}{8}(R^4-r^4)$ $I_y = \frac{\pi}{8}(R^4-r^4)$ $I_{y1} = \frac{\pi(R^2-r^2)(5R^2+r^2)}{8}$ $I_p = I_x + I_y$ $I_{p1} = I_{x_1} + I_{y_1}$
RADIUS OF GYRATION	$k_x = 0.264R = 0.132D$ $k_{x_1} = 0.5R = 0.25D$ $k_y = 0.5R = 0.25D$ $k_{y_1} = 1.118R = 0.559D$ $k_p = 0.566R = 0.2828D$ $k_{p_1} = 1.225R = 0.6124D$	$k_x = \sqrt{\frac{2I_x}{\pi(R^2 - r^2)}} \quad k_{y_1} = \sqrt{\frac{2I_{y1}}{\pi(R^2-r^2)}}$ $k_{x_1} = \sqrt{\frac{R^2 + r^2}{2}} \quad k_p = \sqrt{\frac{2I_p}{\pi(R^2-r^2)}}$ $k_y = \sqrt{\frac{R^2 + r^2}{2}} \quad k_{p_1} = \sqrt{\frac{2I_{p1}}{\pi(R^2-r^2)}}$

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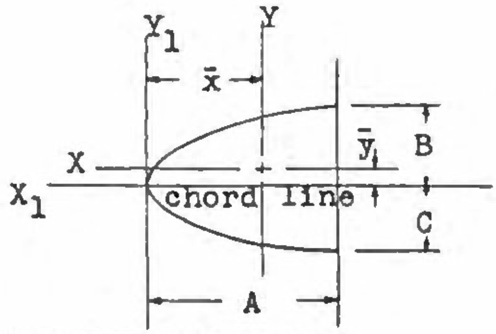
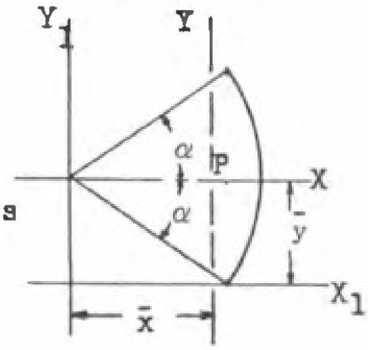
	NOSE RIB	CIRCULAR SECTOR
	 based on parabolic segment	α in radians 
AREA	$\frac{2}{3} A (B + C)$	$R^2 \alpha$
CENTROID	$\bar{x} = 0.6A$ $\quad \bar{y} = 0.375 (B-C)$	$\bar{x} = \frac{2}{3}\left[\frac{R \sin \alpha}{\alpha}\right]$ $\quad \bar{y} = R \sin \alpha$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{W}{320}(19B^2 + 26BC + 19C^2)$ $I_{x1} = \frac{W}{5}(B^2 - BC + C^2)$ $I_y = 0.0686WA^2$ $I_{y1} = 0.4286WA^2$ $I_p = I_x + I_y$ $I_{p1} = I_{x1} + I_{y1}$	$I_x = \frac{WR^2}{4\alpha}(\alpha - \sin\alpha \cos\alpha)$ $I_{x1} = I_x + WR^2 \sin^2\alpha$ $I_y = \frac{WR^2}{4\alpha}\left(\alpha - \frac{16 \sin^2\alpha}{9\alpha} + \frac{\sin 2\alpha}{2}\right)$ $I_{y1} = \frac{WR^2}{4\alpha}(\alpha + \sin\alpha \cos\alpha)$ $I_p = I_x + I_y$
AREA MOMENT OF INERTIA	$I_x = \frac{A(B + C)}{480}(19B^2 + 26BC + 19C^2)$ $I_{x1} = 0.1333(AB + AC)(B^2 - BC + C^2)$ $I_y = 0.0457\, A^3 (B + C)$ $I_{y1} = 0.2857\, A^3 (B + C)$ $I_p = I_x + I_y$ $I_{p1} = I_{x1} + I_{y1}$	$I_x = \frac{R^4}{4}(\alpha - \sin\alpha \cos\alpha)$ $I_{x1} = \frac{R^4}{4}(\alpha - \sin\alpha \cos\alpha) + R^4 \alpha \sin^2\alpha$ $I_y = \frac{R^4}{4}\left(\alpha - \frac{16 \sin^2\alpha}{9\alpha} + \frac{\sin 2\alpha}{2}\right.$ $I_{y1} = \frac{R^4}{4}(\alpha + \sin\alpha \cos\alpha)$ $I_p = \frac{R^4}{4}\left(2\alpha - \frac{16 \sin^2\alpha}{9\alpha}\right)$
RADIUS OF GYRATION	$k_x = \sqrt{\frac{3I_x}{2A(B+C)}}$ $k_y = \sqrt{\frac{3I_y}{2A(B+C)}}$	$k_x = \frac{R}{2}\sqrt{1 - \frac{\sin\alpha \cos\alpha}{\alpha}}$ $k_{x1} = \sqrt{\frac{I_{x1}}{R^2 \alpha}}$ $k_y = \frac{R}{2}\sqrt{1 + \frac{\sin\alpha \cos\alpha}{\alpha} - \frac{16 \sin^2\alpha}{9\alpha^2}}$ $k_{y1} = \frac{R}{2}\sqrt{1 + \frac{\sin\alpha \cos\alpha}{\alpha}}$ $k_p = \frac{R}{2}\sqrt{2 - \frac{16 \sin^2\alpha}{9\alpha^2}}$

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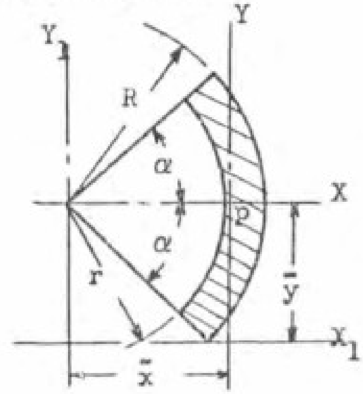
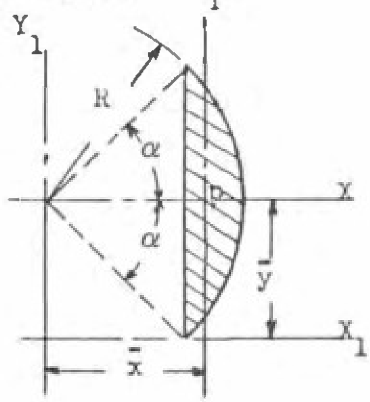
	HOLLOW CIRCULAR SECTOR α in radians 
AREA	$(R^2 - r^2)\alpha$
CENTROID	$\bar{x} = \frac{2\sin\alpha\,(R^3-r^3)}{3\,\alpha\,(R^2-r^2)}$ $\quad \bar{y} = R\sin\alpha$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{W}{4}(R^2+r^2)\left(1-\frac{\sin\alpha\cos\alpha}{\alpha}\right)$ $\quad I_{y_1} = \frac{W}{4}(R^2+r^2)\left(1+\frac{\sin\alpha\cos\alpha}{\alpha}\right)$ $I_{x_1} = I_x + WR^2\sin^2\alpha$ $\quad I_p = I_x + I_y$ $I_y = I_{y_1} - W\left[\frac{2\sin\alpha\,(R^3-r^3)}{3\alpha\,(R^2-r^2)}\right]^2$
AREA MOMENT OF INERTIA	$I_x = \frac{\alpha}{4}(R^4-r^4)\left(1-\frac{\sin\alpha\cos\alpha}{\alpha}\right)$ $\quad I_{y_1} = \frac{\alpha}{4}(R^4-r^4)\left(1+\frac{\sin\alpha\cos\alpha}{\alpha}\right)$ $I_{x_1} = I_x + \alpha\,(R^4-r^2R^2)\sin^2\alpha$ $\quad I_p = I_x + I_y$ $I_y = I_{y_1} - \frac{1}{\alpha(R^2-r^2)}\left[\frac{2\sin\alpha(R^3-r^3)}{3}\right]^2$
RADIUS OF GYRATION	$k_x = \sqrt{\frac{R^2+r^2}{4}\left[1-\frac{\sin\alpha\cos\alpha}{\alpha}\right]}$ $\quad k_{y_1} = \sqrt{\left[\frac{R^2+r^2}{4}\right]\left[1+\frac{\sin\alpha\cos\alpha}{\alpha}\right]}$ $k_{x_1} = \sqrt{\frac{I_{x_1}}{(R^2-r^2)\,\alpha}}$ $\quad k_p = \sqrt{\frac{I_p}{(R^2-r^2)\alpha}}$ $k_y = \sqrt{\frac{I_y}{(R^2-r^2)\,\alpha}}$

Adapted from Weight Handbook, Vol. 1, Society of Aeronautical Weight Engineers, Inc.

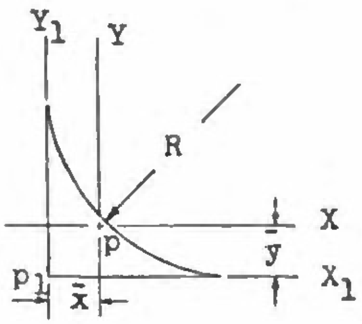
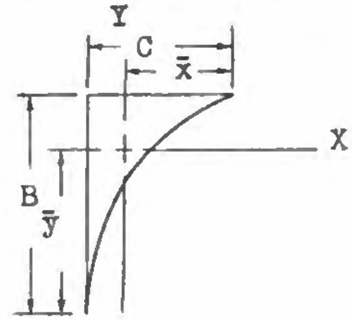
CIRCULAR SEGMENT

α in radians
A = area

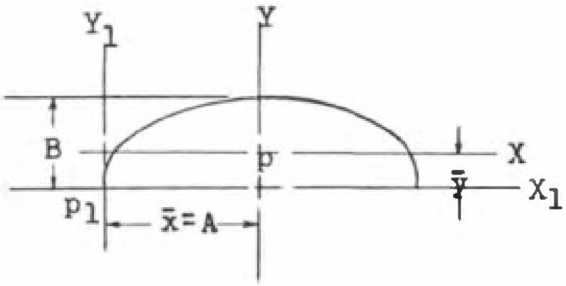
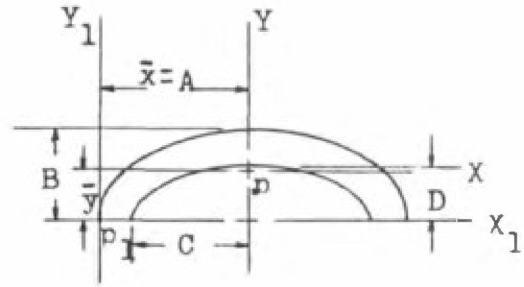


AREA	$\frac{R^2}{2}(2\alpha - \text{SIN } 2\alpha)$
CENTROID	$\bar{x} = \frac{4R\ \text{SIN}^3\alpha}{3(2\alpha - \text{SIN } 2\alpha)}$ $\quad \bar{y} = R\text{SIN } \alpha$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{WR^2}{4}\left[1 - \frac{2\ \text{SIN}^3\alpha\ \text{COS } \alpha}{3(\alpha - \text{SIN } \alpha\ \text{COS } \alpha)}\right]$ $\quad I_{y_1} = \frac{WR^2}{4}\left[1 + \frac{2\ \text{SIN}^3\alpha\ \text{COS } \alpha}{\alpha - \text{SIN } \alpha\ \text{COS } \alpha}\right]$ $I_{x_1} = I_x + WR^2\text{SIN}^2\alpha$ $\quad I_p = I_x + I_y$ $I_y = I_{y_1} - W\left[\frac{2R\ \text{SIN}^3\alpha}{3(\alpha - \text{SIN } \alpha\ \text{COS } \alpha)}\right]^2$
AREA MOMENT OF INERTIA	$I_x = \frac{AR^2}{4}\left[1 - \frac{2}{3}\left[\frac{\text{SIN}^3\alpha\ \text{COS } \alpha}{\alpha - \text{SIN } \alpha\ \text{COS } \alpha}\right]\right]$ $\quad I_y = \frac{AR^2}{4}\left[1 + \frac{2\ \text{SIN}^3\alpha\ \text{COS } \alpha}{\alpha - \text{SIN } \alpha\ \text{COS } \alpha}\right]$ $I_{x_1} = I_x + \frac{R^4}{2}(2\alpha - \text{SIN } 2\alpha)(\text{SIN}^2\alpha)$ $\quad I_p = I_x + I_y$ $I_y = I_{y_1} - \frac{4R^6\ \text{SIN}^6\alpha}{9A}$
RADIUS OF GYRATION	$k_x = \sqrt{\frac{R^2}{4}\left[1 - \frac{2\ \text{SIN}^3\alpha\ \text{COS } \alpha}{3(\alpha - \text{SIN } \alpha\ \text{COS } \alpha)}\right]}$ $k_y = \sqrt{\frac{2I_y}{R^2(2\alpha - \text{SIN}2\alpha)}}$ $k_p = \sqrt{\frac{2I_p}{R^2(2\alpha - \text{SIN } 2\alpha)}}$

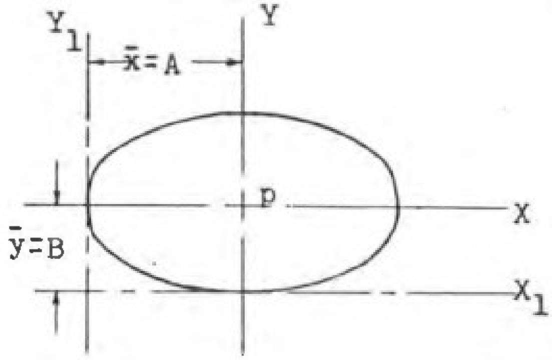
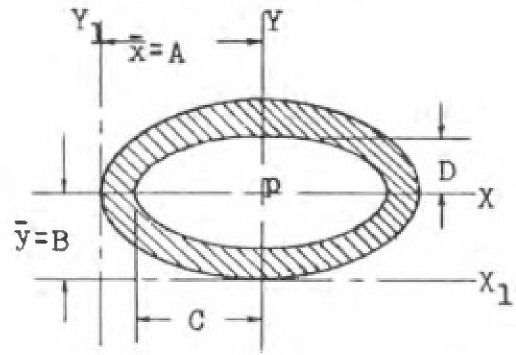
Adapted from Weight Handbook, Vol. 1, Society of Aeronautical Weight Engineers, Inc.

	CIRCULAR COMPLEMENT	ELLIPTIC COMPLEMENT
		
AREA	$0.2146\ R^2$	$0.2146\ BC$
CENTROID	$\bar{x} = \bar{y} = 0.2234\ R$	$\bar{x} = \frac{C}{1.288} = 0.7766C$ $\bar{y} = \frac{B}{1.288} = 0.7766B$
WEIGHT MOMENT OF INERTIA	$I_x = I_y = 0.035\ WR^2$ $I_{x_1} = I_{y_1} = 0.085\ WR^2$ $I_p = 0.070\ WR^2$ $I_{p_1} = 0.170\ WR^2$	$I_x = 0.035\ WB^2$ $I_y = 0.035\ WC^2$ $I_p = 0.035\ W(B^2 + C^2)$
AREA MOMENT OF INERTIA	$I_x = I_y = 0.0075\ R^4$ $I_{x_1} = I_{y_1} = 0.0183\ R^4$ $I_p = 0.0151\ R^4$ $I_{p_1} = 0.0365\ R^4$	$I_x = 0.00758^3C$ $I_y = 0.0075BC^3$ $I_p = 0.0075\ BC(B^2 + C^2)$
RADIUS OF GYRATION	$k_x = k_y = 0.188\ R$ $k_{x_1} = k_{y_1} = 0.292\ R$ $k_p = 0.265\ R$ $k_{p_1} = 0.412\ R$	$k_x = 0.1888$ $k_y = 0.188C$ $k_p = \sqrt{0.035(B^2 + C^2)}$

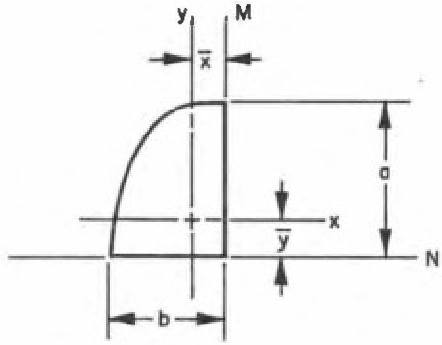
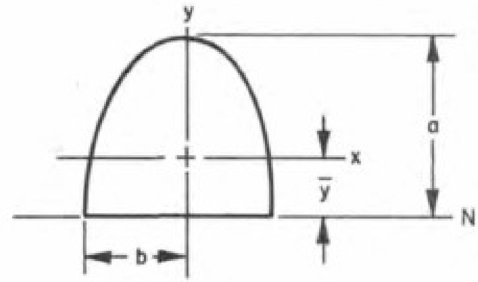
Adapted from Weight Handbook, Vol. 1, Society of Aeronautical Weight Engineers, Inc.

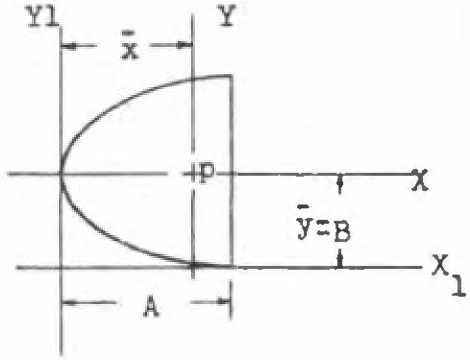
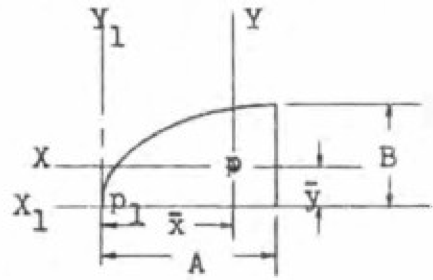
	SEMIELLIPSE	HOLLOW SEMIELLIPSE
		
AREA	$\frac{\pi AB}{2}$	$\frac{\pi(AB-CD)}{2}$
CENTROID	$\bar{x} = A$ $\bar{y} = 0.424\ B$	$\bar{x} = A$ $\bar{y} = \frac{4}{3\pi}\left[\frac{AB^2-CD^2}{AB-CD}\right]$
WEIGHT MOMENT OF INERTIA	$I_x = 0.070\ WB^2$ $I_{x_1} = 0.25\ WB^2$ $I_y = 0.25\ WA^2$ $I_{y_1} = 1.25\ WA^2$ $I_p = \frac{W(A^2 + 0.280B^2)}{4}$ $I_{p_1} = \frac{W(5A^2 + B^2)}{4}$	$I_x = \frac{W}{4}\left(\frac{AB^3-CD^3}{AB-CD}\right) - W\left[\frac{4}{3\pi}\left(\frac{AB^2-CD^2}{AB-CD}\right)\right]^2$ $I_{x_1} = \frac{W}{4}\left(\frac{AB^3-CD^3}{AB-CD}\right)$ $I_y = \frac{W}{4}\left(\frac{A^3B-C^3D}{AB-CD}\right)$ $I_{y_1} = \frac{W}{4}\left(\frac{A^3B-C^3D}{AB-CD}\right) + WA^2$ $I_p = I_x + I_y$
AREA MOMENT OF INERTIA	$I_x = 0.1098\ AB^3$ $I_{x_1} = 0.3927\ AB^3$ $I_y = 0.3927\ A^3B$ $I_{y_1} = 1.9635\ A^3B$ $I_p = AB\ (0.3927\ A^2 + 0.1098\ B^2)$ $I_{p_1} = AB\ (1.9635\ A^2 + 0.3927\ B^2)$	$I_x = \frac{\pi}{8}(AB^3-CD^3) - \frac{\pi(AB-CD)}{2}\left[\frac{4(AB^2-CD^2)}{3\pi(AB-CD)}\right]^2$ $I_{x_1} = \frac{\pi}{8}(AB^3-CD^3)$ $I_y = \frac{\pi}{8}(A^3B-C^3D)$ $I_{y_1} = \frac{\pi}{8}(A^3B-C^3D) + \frac{\pi A^2(AB-CD)}{2}$ $I_p = I_x + I_y$ $I_{p_1} = I_{x_1} + I_{y_1}$
RADIUS OF GYRATION	$k_x = 0.2643\ B$ $k_{x_1} = \frac{B}{2}$ $k_y = \frac{A}{2}$ $k_{y_1} = 1.118\ A$ $k_p = \sqrt{\frac{A^2}{4} + \frac{B^2}{4} - \frac{16\ B^2}{9\pi^2}}$ $k_{p_1} = \sqrt{\frac{2I_{p_1}}{\pi AB}}$	$k_x = \sqrt{\frac{2I_x}{\pi(AB-CD)}}$ $k_{x_1} = \sqrt{\frac{AB^3-CD^3}{4(AB-CD)}}$ $k_y = \sqrt{\frac{2I_y}{\pi(AB-CD)}}$ $k_{y_1} = \sqrt{\frac{2I_{y_1}}{\pi(AB-CD)}}$ $k_p = \sqrt{\frac{2I_p}{\pi(AB-CD)}}$ $k_{p_1} = \sqrt{\frac{2I_{p_1}}{\pi(AB-CD)}}$

Adapted from Weight Handbook, Vol. 1, Society of Aeronautical Weight Engineers, Inc.

	ELLIPSE	HOLLOW ELLIPSE
		
AREA	πAB	$\pi(AB-CD)$
CENTROID	$\bar{x} = A$ $\quad \bar{y} = B$	$\bar{x} = A$ $\quad \bar{y} = B$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{WB^2}{4}$ $I_{x_1} = 1.25\ WB^2$ $I_y = \frac{WA^2}{4}$ $I_{y_1} = 1.25\ WA^2$ $I_p = \frac{W(A^2+B^2)}{4}$	$I_x = \frac{W}{4}\left[\frac{AB^3-CD^3}{AB-CD}\right]$ $I_{x_1} = \frac{W}{4}\left[\frac{AB^3-CD^3}{AB-CD}\right] + WB^2$ $I_y = \frac{W}{4}\left[\frac{A^3B-C^3D}{AB-CD}\right]$ $I_{y_1} = \frac{W}{4}\left[\frac{A^3B-C^3D}{AB-CD}\right] + WA^2$ $I_p = I_x + I_y$
AREA MOMENT OF INERTIA	$I_x = \frac{\pi AB^3}{4} = 0.7854\ AB^3$ $I_{x_1} = 1.25\ \pi AB^3 = 3.927\ AB^3$ $I_y = \frac{\pi A^3B}{4} = 0.7854\ A^3B$ $I_{y_1} = 1.25\ \pi A^3B = 3.927\ A^3B$ $I_p = \frac{\pi AB(A^2+B^2)}{4}$	$I_x = \frac{\pi}{4}(AB^3-CD^3)$ $I_{x_1} = \frac{\pi}{4}(AB^3-CD^3) + \pi(AB-CD)(B^2)$ $I_y = \frac{\pi}{4}(A^3B - C^3D)$ $I_{y_1} = \frac{\pi}{4}(A^3B-C^3D) + \pi(AB-CD)(A^2)$ $I_p = I_x + I_y$
RADIUS OF GYRATION	$k_x = \frac{B}{2}$ $k_{x_1} = 1.118\ B$ $k_y = \frac{A}{2}$ $k_{y_1} = 1.118\ A$ $k_p = \frac{\sqrt{A^2+B^2}}{2}$	$k_x = \sqrt{\frac{AB^3-CD^3}{4(AB-CD)}}$ $k_{x_1} = \sqrt{\frac{I_{x_1}}{\pi(AB-CD)}}$ $k_y = \sqrt{\frac{A^3B-C^3D}{4(AB-CD)}}$ $k_{y_1} = \sqrt{\frac{I_{y_1}}{\pi(AB-CD)}}$ $k_p = \sqrt{\frac{I_p}{\pi(AB-CD)}}$

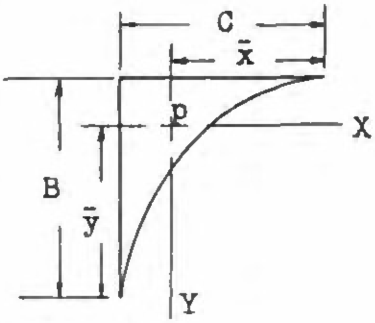
Adapted from Weight Handbook, Vol. 1, Society of Aeronautical Weight Engineers, Inc.

	QUARTER ELLIPSE	HALF ELLIPSE
		
AREA	$\pi ab/4$	$\pi ab/2$
CENTROID	$\bar{y} = \frac{4a}{3\pi}$ $\bar{x} = \frac{4b}{3\pi}$	$\bar{y} = \frac{4a}{3\pi}$ $\bar{x} = 0$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{4ma}{\pi}\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)$ $I_y = \frac{4mb^2}{\pi}\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)$ $I_N = \frac{ma^2}{4}$ $I_M = \frac{mb^2}{4}$	$I_x = \frac{2ma^2}{\pi}\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)$ $I_y = \frac{mb^2}{4}$ $I_N = \frac{ma^2}{4}$
AREA MOMENT OF INERTIA	$I_x = a^3b\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)$ $I_y = ab^3\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)$ $I_N = \frac{\pi a^3 b}{16}$ $I_M = \frac{\pi ab^3}{16}$	$I_x = a^3b\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)$ $I_y = \frac{\pi ab^3}{8}$ $I_N = \frac{\pi a^3 b}{8}$
RADIUS OF GYRATION	$k_x = \sqrt{I_x/A}$ $k_y = \sqrt{I_y/A}$ $k_N = \sqrt{I_N/A}$ $k_M = \sqrt{I_M/A}$	$k_x = \sqrt{I_x/A}$ $k_y = \sqrt{I_y/A}$ $k_N = \sqrt{I_N/A}$

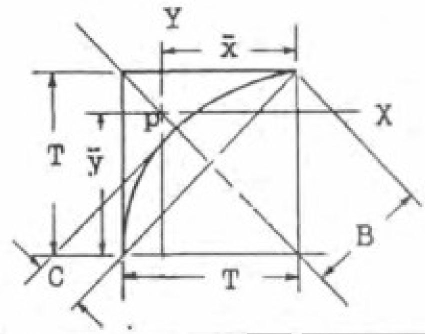
	PARABOLIC SEGMENT 	PARABOLIC HALF-SEGMENT 
AREA	$\frac{4}{3} AB$	$\frac{2AB}{3}$
CENTROID	$\bar{x} = 0.6A$ $\bar{y} = B$	$\bar{x} = 0.6A$ $\bar{y} = 0.375B$
WEIGHT MOMENT OF INERTIA	$I_x = 0.2WB^2$ $I_{x_1} = 1.2\ WB^2$ $I_y = 0.0686WA^2$ $I_{y_1} = 0.4286WA^2$ $I_p = I_x + I_y$	$I_x = 0.0594WB^2$ $I_{x_1} = 0.2WB^2$ $I_y = 0.0686WA^2$ $I_{y_1} = 0.4286WA^2$ $I_p = I_x + I_y$ $I_{p_1} = I_{x_1} + I_{y_1}$
AREA MOMENT OF INERTIA	$I_x = 0.2667AB^3$ $I_{x_1} = 1.6AB^3$ $I_y = 0.0914A^3B$ $I_{y_1} = 0.5714A^3B$ $I_p = I_x + I_y$	$I_x = 0.0396AB^3$ $I_{x_1} = 0.1333AB^3$ $I_y = 0.0457A^3B$ $I_{y_1} = 0.2857A^3B$ $I_p = I_x + I_y$ $I_{p_1} = I_{x_1} + I_{y_1}$
RADIUS OF GYRATION	$k_x = 0.4472B$ $k_{x_1} = 1.095B$ $k_y = 0.2619A$ $k_{y_1} = 0.6547A$ $k_p = \sqrt{\frac{3I_p}{4AB}}$	$k_x = 0.2437B$ $k_{x_1} = 0.4472B$ $k_y = 0.2619A$ $k_{y_1} = 0.6547A$ $k_p = \sqrt{\frac{3I_p}{2AB}}$ $k_{p_1} = \sqrt{\frac{3I_{p_1}}{2AB}}$

Adapted from Weight Handbook, Vol. I, Society of Aeronautical Weight Engineers, Inc.

COMPLEMENT OF HALF PARABOLA



PARABOLIC FILLET IN RIGHT ANGLE

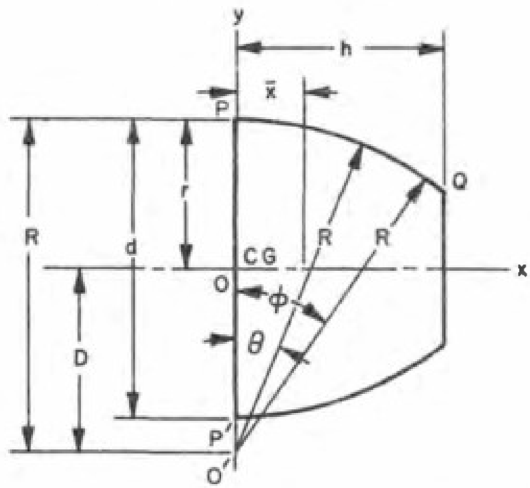


	COMPLEMENT OF HALF PARABOLA	PARABOLIC FILLET IN RIGHT ANGLE
AREA	$\frac{1}{3}\ BC$	$\frac{T^2}{6}$
CENTROID	$\bar{x} = \frac{3}{4}C \qquad \bar{y} = \frac{7}{10}B$	$\bar{x} = \bar{y} = \frac{4}{5}T \quad B = \frac{T}{\sqrt{2}} \quad C = \frac{T}{2\sqrt{2}}$
WEIGHT MOMENT OF INERTIA	$I_x = \frac{37}{700}WB^2$ $I_y = \frac{3}{80}WC^2$ $I_p = \frac{W}{20}\left(\frac{37}{35}B^2 + \frac{3C^2}{4}\right)$	$I_x = I_y = \frac{3WT^2}{100}$ $I_p = \frac{3WT^2}{50}$
AREA MOMENT OF INERTIA	$I_x = \frac{37}{2100}B^3C$ $I_y = \frac{1}{80}BC^3$ $I_p = \frac{BC}{20}\left(\frac{37}{105}B^2 + \frac{C^2}{4}\right)$	$I_x = I_y = \frac{11}{2100}T^4$ $I_p = \frac{T^4}{100}$
RADIUS OF GYRATION	$k_x = 0.230\ B$ $k_y = 0.194\ C$ $k_p = \sqrt{\frac{3I_p}{BC}}$	$k_x = k_y = 0.173\ T$ $k_p = 0.2449\ T$

Adapted from Weight Handbook, Vol. 1, Society of Aeronautical Weight Engineers, Inc.

OGIVAL SHAPES

An ogival shape is one that is developed as a convex solid of revolution.



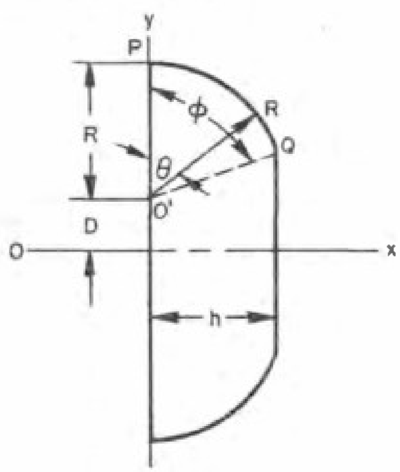
The solid of revolution developed in the diagram above is bounded by the arc PQ of a circle of radius R (radius of longitudinal curvature) whose center, o′, lies on the side of the axis of revolution, ox, opposite to the arc PQ and distant D = oo′ from the axis of revolution ox.

If the arc PQ cuts the axis ox for the desired length, h, the ogive is said to be pointed. The diameter of the base, d, is known as the diameter of swell, which is effectively the maximum transverse diameter of the body.

A tangent ogive is one that includes its base as illustrated in the above diagram.

A secant ogive is one that does not include its base; that is, the length h does not extend to the point where the maximum swell occurs.

If the radial center of the arc PQ, designated as o′, is moved to a location on the arc (PQ) side of the axis of revolution ox, a non-standard ogive will result as illustrated below.



The properties of ogival shapes are included because of their common usage in the design of missiles, artillery shells, and other systems requiring this family of geometrical shapes.

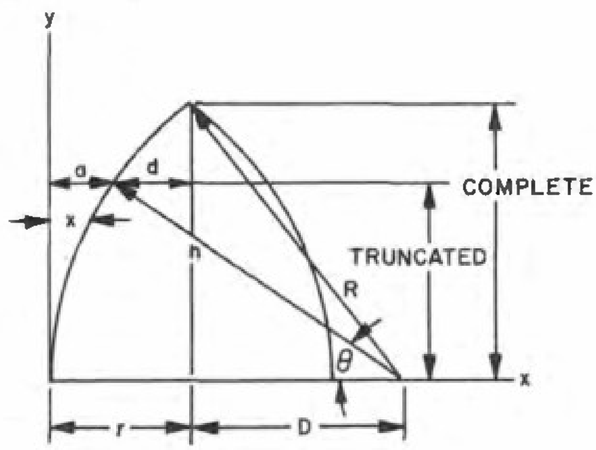
Sample derivations are given so that the reader can quickly review methods of approach in the basic steps involved in the development of a general equation used to describe an ogival property.

The section entitled Properties of a Solid Ogive contains the equations describing the volume, moment, and moments of inertia about the central axis and the base plane for both the truncated and the complete ogive.

These equations have been developed in both the exact and approximate forms. Examination of the equations will show that the term $\theta/\sin\theta$ found in the exact equations has been replaced by a convergent series expansion, thereby leading to the simpler approximate forms.

The use of the approximate equations is left to the reader's discretion; however, tables of expected error are included as a guide in determining the number of terms to be used in the equations.

PROPERTIES OF A SOLID OGIVE



R = ogive radius

h = length of ogival head (truncated or complete)

r = radius of base

d = radius of truncated nose ($d = 0$ for complete ogive)

$D = R - r$

$a = r - d$

$\sin\theta = h/R$

SUMMARY OF EQUATIONS FOR A TRUNCATED OGIVE

Volume

$$V = \pi h\left[R^2 - \frac{h^2}{3} - D\left(d + R\,\frac{\theta}{\sin\theta}\right)\right]$$

$$V = \frac{\pi h}{9}\left\{(3r - a)^2 + 2a^2 - 6a^2\frac{R - r}{R}\left[\frac{1}{5} + \frac{3}{35}\frac{a}{R} + \frac{4}{105}\left(\frac{a}{R}\right)^2 + \frac{4}{231}\left(\frac{a}{R}\right)^3\right.\right.$$

$$\left.\left. + \frac{8}{1,001}\left(\frac{a}{R}\right)^4 + \frac{8}{2,145}\left(\frac{a}{R}\right)^5 + \frac{64}{36,465}\left(\frac{a}{R}\right)^6 + \frac{192}{230,945}\left(\frac{a}{R}\right)^7 + \cdots\right]\right\}$$

$$= \frac{\pi h a^2}{9}\left\{\left(\frac{3r}{a} - 1\right)^2 + 2 - 6\left(1 - \frac{r}{R}\right)\left[\frac{1}{5} + \frac{3}{35}\frac{a}{R} + \frac{4}{105}\left(\frac{a}{R}\right)^2 + \frac{4}{231}\left(\frac{a}{R}\right)^3\right.\right.$$

$$\left.\left. + \frac{8}{1,001}\left(\frac{a}{R}\right)^4 + \frac{8}{2,145}\left(\frac{a}{R}\right)^5 + \frac{64}{36,465}\left(\frac{a}{R}\right)^6 + \frac{192}{230,945}\left(\frac{a}{R}\right)^7 + \cdots\right]\right\}$$

First Moment

$$\bar{y}V = \pi\left[\frac{h^2}{2}\left(r^2 + \frac{2}{3}RD - \frac{h^2}{2}\right) - \frac{2}{3}Da(R - a)^2\right]$$

$$= \pi a\left[Rrd + \frac{1}{3}a^2(R + 2r) - \frac{1}{2}a\left(r^2 + \frac{1}{2}a^2\right)\right]$$

Moment of Inertia About the Central Axis

$$I_A = \frac{\pi}{2}h\left[(R^2 + D^2)^2 + 2D^2(2R^2 - h^2) - h^2\left(\frac{2}{3}R^2 - \frac{1}{5}h^2\right)\right]$$

$$- \frac{\pi}{4}Dh\left[\left(R\frac{\theta}{\sin\theta} + R - a\right)(4D^2 + 3R^2) + 2(R - a)^3\right]$$

$$I_A = \frac{\pi}{2}h\left\{r^3\left(r - \frac{4}{3}a\right) + \frac{2}{15}r^2a^2\left(9 + 2\frac{r}{R}\right) + \frac{1}{5}a^3(R - 5r + a)\right.$$

$$- a^4\left[7 - 15\frac{r}{R} + 12\left(\frac{r}{R}\right)^2 - 4\left(\frac{r}{R}\right)^3\right]\left[\frac{1}{35}\frac{R}{a} + \frac{4}{315} + \frac{4}{693}\frac{a}{R} + \frac{8}{3,003}\left(\frac{a}{R}\right)^2\right.$$

$$\left.\left. + \frac{8}{6,435}\left(\frac{a}{R}\right)^3 + \frac{64}{109,395}\left(\frac{a}{R}\right)^4 + \frac{64}{230,945}\left(\frac{a}{R}\right)^5 + \cdots\right]\right\}$$

$$= \frac{\pi}{2}ha^4\left\{\left(\frac{r}{a}\right)^3\left(\frac{r}{a} - \frac{4}{3}\right) + \frac{2}{15}\left(\frac{r}{a}\right)^2\left(9 + 2\frac{r}{R}\right) + \frac{1}{5}\left(\frac{R}{a} - \frac{5r}{a} + 1\right)\right.$$

$$- \left[7 - 15\frac{r}{R} + 12\left(\frac{r}{R}\right)^2 - 4\left(\frac{r}{R}\right)^3\right]\left[\frac{1}{35}\frac{R}{a} + \frac{4}{315} + \frac{4}{693}\frac{a}{R} + \frac{8}{3,003}\left(\frac{a}{R}\right)^2\right.$$

$$\left.\left. + \frac{8}{6,435}\left(\frac{a}{R}\right)^3 + \frac{64}{109,395}\left(\frac{a}{R}\right)^4 + \frac{64}{230,945}\left(\frac{a}{R}\right)^5 + \cdots\right]\right\}$$

Note. Since a density of unity was used in the derivation of the equations, it would be well to recall the relationship $\rho = m/V$. The use of this relationship when coupled with the moment of inertia equations leads to the determination of the mass moment of inertia.

Moment of Inertia About the Base Plane

$$I_B = \frac{\pi}{15}h^3\left[5(R^2 + D^2) - 3h^2\right] - \frac{\pi}{4}Dh\left[R^3\frac{\theta}{\sin\theta} - (R - a)(R^2 - 2h^2)\right] + \frac{1}{2}I_A$$

$$I_B = \pi ha\left\{\frac{2}{7}a^2\left(R + \frac{9}{5}r\right) + \frac{1}{3}r^2(2R - a) - \frac{1}{5}a(4Rr + a^2)\right.$$
$$- 2a^3\left(\frac{R - r}{R}\right)\left[\frac{1}{315} + \frac{1}{693}\frac{a}{R} + \frac{2}{3{,}003}\left(\frac{a}{R}\right)^2 + \frac{2}{6{,}435}\left(\frac{a}{R}\right)^3\right.$$
$$\left.\left. + \frac{16}{109{,}395}\left(\frac{a}{R}\right)^4 + \frac{16}{230{,}945}\left(\frac{a}{R}\right)^5 + \cdots\right]\right\} + \frac{1}{2}I_A$$

$$= \pi ha^4\left\{\frac{2}{7}\left(\frac{R}{a} + \frac{9}{5}\frac{r}{a}\right) + \frac{1}{3}\left(\frac{r}{a}\right)^2\left(\frac{2R}{a} - 1\right) - \frac{1}{5}\left(\frac{4Rr}{a^2} + 1\right)\right.$$
$$- 2\left(1 - \frac{r}{R}\right)\left[\frac{1}{315} + \frac{1}{693}\frac{a}{R} + \frac{2}{3{,}003}\left(\frac{a}{R}\right)^2 + \frac{2}{6{,}435}\left(\frac{a}{R}\right)^3\right.$$
$$\left.\left. + \frac{16}{109{,}395}\left(\frac{a}{R}\right)^4 + \frac{16}{230{,}945}\left(\frac{a}{R}\right)^5 + \cdots\right]\right\} + \frac{1}{2}I_A$$

SUMMARY OF EQUATIONS FOR A COMPLETE OGIVE

Volume

$$V = \pi h\left[R^2 - \frac{h^2}{3} - DR\frac{\theta}{\sin\theta}\right]$$

$$V = \frac{2}{3}\pi hr^2\left[\frac{4}{5} + \frac{4}{35}\frac{r}{R} + \frac{1}{21}\left(\frac{r}{R}\right)^2 + \frac{8}{385}\left(\frac{r}{R}\right)^3 + \frac{4}{429}\left(\frac{r}{R}\right)^4 + \frac{64}{15{,}015}\left(\frac{r}{R}\right)^5\right.$$
$$\left. + \frac{24}{12{,}155}\left(\frac{r}{R}\right)^6 + \frac{128}{138{,}567}\left(\frac{r}{R}\right)^7 + \frac{64}{146{,}965}\left(\frac{r}{R}\right)^8 + \cdots\right]$$

First Moment

$$\bar{y}V = \frac{\pi}{3}r^3\left(R - \frac{r}{4}\right)$$

Moment of Inertia About the Central Axis

$$I_A = \frac{\pi}{2} h \left[R^2 \left(R^2 + \frac{5}{2} D^2 \right) - h^2 \left(\frac{2}{3} D^2 + \frac{7}{15} h^2 \right) - \frac{RD}{2} \frac{\theta}{\sin \theta} (4D^2 + 3R^2) \right]$$

$$I_A = 4\pi h r^4 \left[\frac{16}{315} + \frac{32}{3,465} \frac{r}{R} + \frac{8}{2,145} \left(\frac{r}{R}\right)^2 + \frac{8}{5,005} \left(\frac{r}{R}\right)^3 + \frac{181}{255,255} \left(\frac{r}{R}\right)^4 + \frac{8}{24,871} \left(\frac{r}{R}\right)^5 + \frac{28}{188,955} \left(\frac{r}{R}\right)^6 + \cdots \right]$$

Moment of Inertia About the Base Plane

$$I_B = \frac{\pi}{60} h^3 (8R^2 + 17D^2) - \frac{\pi}{4} Dh \left(R^3 \frac{\theta}{\sin \theta} - D^3 \right) + \frac{1}{2} I_A$$

$$I_B = 2\pi h r^4 \left[\frac{8}{105} \frac{R}{r} - \frac{4}{315} + \frac{2}{1,155} \frac{r}{R} + \frac{1}{1,287} \left(\frac{r}{R}\right)^2 + \frac{16}{45,045} \left(\frac{r}{R}\right)^3 + \frac{2}{12,155} \left(\frac{r}{R}\right)^4 + \frac{32}{415,701} \left(\frac{r}{R}\right)^5 + \frac{16}{440,895} \left(\frac{r}{R}\right)^6 + \frac{128}{7,436,429} \left(\frac{r}{R}\right)^7 + \cdots \right] + \frac{1}{2} I_A$$

TABLE 2. UPPER BOUND FOR ERROR WITH THE USE OF APPROXIMATE EQUATIONS FOR THE VOLUME OF A TRUNCATED OGIVE

Number of terms	Upper bound for error, %			
	a/r = 0.3	a/r = 0.5	a/r = 0.7	a/r = 0.9
r/R = 0.1				
None	1.3	4.1	8.8	15.2
One	0.02	0.09	0.3	0.7
Two	......	......	0.009	0.03
r/R = 0.3				
None	1.1	3.4	7.3	13.1
One	0.04	0.2	0.7	1.7
Two	......	0.02	0.07	0.2
Three....	......	......	......	0.03
r/R = 0.5				
None	0.8	2.5	5.7	10.4
One	0.05	0.3	0.9	2.2
Two	......	0.03	0.2	0.4
Three....	......	......	0.02	0.09
r/R = 0.7				
None	0.5	1.6	3.7	7.1
One	0.04	0.3	0.8	2.0
Two	......	0.04	0.2	0.6
Three....	......	......	0.04	0.2
Four......	......	......	......	0.05

EXPECTED ERROR WITH THE USE OF APPROXIMATE EQUATIONS

Tables 2–7 give the upper bounds for error incurred with the use of the approximate equations. Examination of the tables will show that including a sufficient number of terms in the convergent series expansion will reduce the error markedly.

TABLE 3. UPPER BOUND FOR ERROR WITH THE USE OF APPROXIMATE EQUATIONS FOR THE MOMENT OF INERTIA ABOUT THE CENTRAL AXIS FOR A TRUNCATED OGIVE

Number of terms	Upper bound for error, %			
	a/r = 0.3	a/r = 0.5	a/r = 0.7	a/r = 0.9
		r/R = 0.1		
None	6.0	26.5	53.9	74.1
One	0.08	0.8	3.5	10.1
Two	......	0.02	0.1	0.5
Three....	......	......	0.004	0.02
		r/R = 0.3		
None	1.32	7.1	20.2	38.4
One	0.05	0.5	2.3	6.9
Two	......	0.03	0.2	0.9
Three....	......	......	0.02	0.1
Four......	......	......	......	0.02
		r/R = 0.5		
None	0.5	2.7	8.5	18.7
One	0.03	0.3	1.4	4.4
Two	......	0.04	0.2	0.9
Three....	......	......	0.04	0.2
Four......	......	......	......	0.04
		r/R = 0.7		
None	0.2	1.0	3.4	8.2
One	0.02	0.2	0.8	2.4
Two	......	0.03	0.2	0.7
Three....	......	......	0.04	0.2
Four......	......	......	......	0.05

TABLE 4. UPPER BOUND FOR ERROR WITH THE USE OF APPROXIMATE EQUATIONS FOR THE MOMENT OF INERTIA ABOUT THE BASE PLANE FOR A TRUNCATED OGIVE

Number of terms	Upper bound for error, %			
	a/r = 0.3	a/r = 0.5	a/r = 0.7	a/r = 0.9
		r/R = 0.1		
None	0.004	0.02	0.08	0.3
One	0.00005	0.0005	0.003	0.01
		r/R = 0.3		
None	0.01	0.06	0.2	0.7
One	0.0004	0.004	0.02	0.08
		r/R = 0.5		
None	0.01	0.07	0.3	0.9
One	0.0007	0.008	0.05	0.2
Two	......	......	......	0.04
		r/R = 0.7		
None	0.01	0.07	0.3	0.9
One	0.001	0.01	0.06	0.3
Two	......	......	......	0.08

TABLE 5. UPPER BOUND FOR ERROR WITH THE USE OF APPROXIMATE EQUATIONS FOR THE VOLUME OF A COMPLETE OGIVE

Number of terms	Upper bound for error, %			
	r/R = 0.1	r/R = 0.3	r/R = 0.5	r/R = 0.7
One	1.5	5.1	9.5	15.4
Two	0.06	0.6	1.9	4.1
Three	0.003	0.08	0.4	1.2
Four......	......	0.01	0.09	0.4
Five......	......	......	0.002	0.1
Six	......	......	......	0.04

TABLE 6. UPPER BOUND FOR ERROR WITH THE USE OF APPROXIMATE EQUATIONS FOR THE MOMENT OF INERTIA ABOUT THE CENTRAL AXIS FOR A COMPLETE OGIVE

Number of terms	Upper bound for error, %			
	r/R = 0.1	r/R = 0.3	r/R = 0.5	r/R = 0.7
One	1.9	6.4	12.1	19.6
Two	0.08	0.7	2.2	4.9
Three	0.003	0.1	0.5	1.4
Four......	......	0.01	0.1	0.5
Five......	......	......	0.03	0.2
Six	......	......	......	0.05

TABLE 7. UPPER BOUND FOR ERROR WITH THE USE OF APPROXIMATE EQUATIONS FOR THE MOMENT OF INERTIA ABOUT THE BASE PLANE FOR A COMPLETE OGIVE

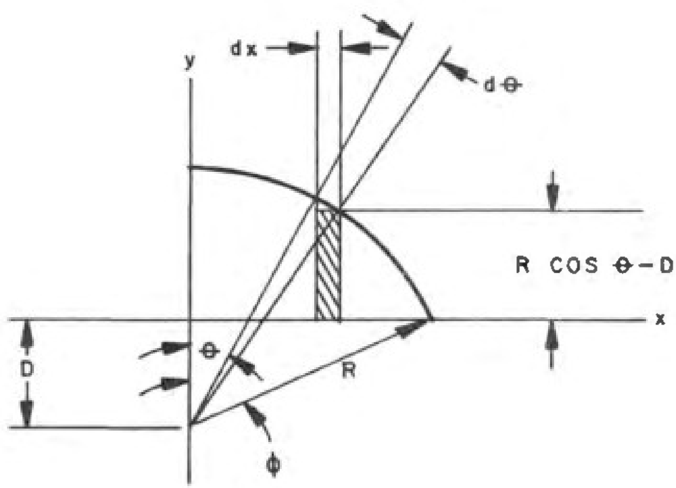
Number of terms	Upper bound for error, %			
	r/R = 0.1	r/R = 0.3	r/R = 0.5	r/R = 0.7
One	1.7	5.0	8.3	11.7
Two	0.03	0.3	0.8	1.9
Three	0.001	0.03	0.2	0.6
Four	......	0.005	0.04	0.2
Five	......	......	0.01	0.06
Six	......	......	......	0.02

ALTERNATIVE EQUATIONS FOR VOLUME, MOMENT, AND MOMENT OF INERTIA OF AN OGIVE

Another and perhaps more concise set of methods for determining the volume, moment, and moment of inertia of an ogival shape is shown below. Again, complete derivations are included that can be used as guidelines in equation development. Special note should be made that angles are measured in radians.

Complete Ogive

Volume.



$$m = D/R$$

$$a = \sin \phi = \sqrt{1 - m^2}$$

$$V = \pi \int y^2 dx$$

$$V = \pi \int (R \cos \theta - D)^2 dx$$

$$= \pi \int (\cos \theta - D/R)^2 R^2 dx$$

$$= \pi \int R^2 (\cos \theta - m)^2 dx$$

$$dx = R(\cos \theta) d\theta$$

$$V = \pi\int R^2(\cos\theta - m)^2 R\cos\theta d\theta$$

$$= \pi R^3\int_0^\phi (\cos\theta - m)^2 \cos\theta d\theta$$

$$= \pi R^3\int_0^\phi (\cos^2\theta - 2m\cos\theta + m^2)\cos\theta d\theta$$

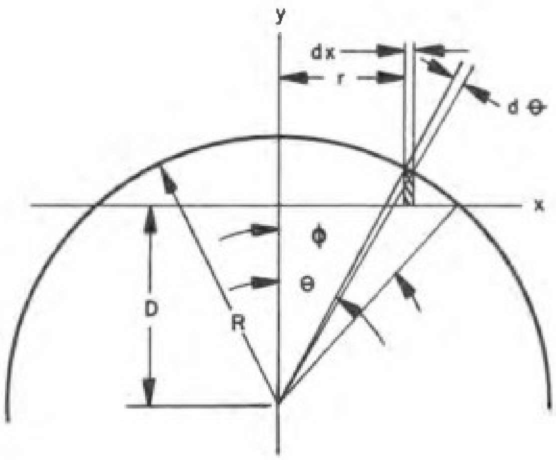
$$= \pi R^3\int_0^\phi (\cos^3\theta - 2m\cos^2\theta + m^2\cos\theta) d\theta$$

$$= \pi R^3\left[\sin\theta - \frac{\sin^3\theta}{3} - 2m\left(\frac{1}{2}\theta + \frac{1}{2}\sin\theta\cos\theta\right) + m^2\sin\theta\right]_0^\phi$$

$$= \pi R^3\left(-\frac{\sin^3\phi}{3} + \sin\phi + m^2\sin\phi - m\sin\phi\cos\phi - m\phi\right)$$

$$= \pi R^3\left(-\frac{a^3}{3} + a - m\phi\right)$$

Moment.



$$m = D/R$$

$$dV = \pi R^3(\cos\theta - m)^2(\cos\theta)d\theta$$

$$r = R\sin\theta$$

$$M = \int r dV$$

$$M = \int_0^\phi (R\sin\theta)\left[\pi R^3(\cos\theta - m)^2(\cos\theta)\right]d\theta$$

$$= \pi R^4\int_0^\phi (\cos^3\theta - 2m\cos^2\theta + m\cos\theta)\sin\theta d\theta$$

$$= \pi R^4\left[-\frac{\cos^4\theta}{4} + \frac{2m\cos^3\theta}{3} - \frac{m^2\cos^2\theta}{2} + \frac{1}{4} - \frac{2m}{3} + \frac{m^2}{2}\right]_0^\phi$$

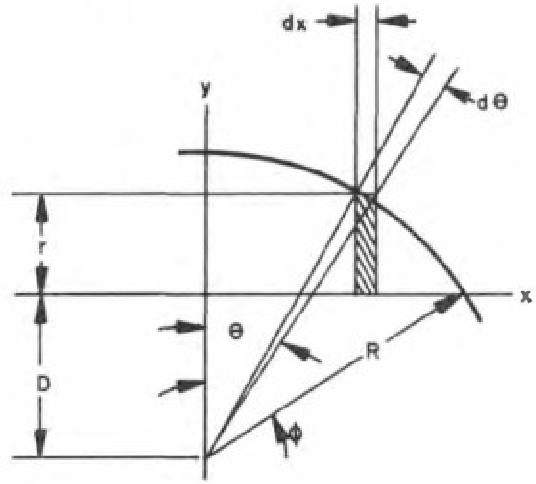
$$m = D/R = \cos\theta$$

$$M = \pi R^4\left(-\frac{m^4}{4} + \frac{2m^4}{3} - \frac{m^4}{2} + \frac{1}{4} - \frac{2m}{3} + \frac{m^2}{2}\right)$$

$$= \pi R^4\left(\frac{-3m^4 + 8m^4 - 6m^4}{12} + \frac{m^2}{2} - \frac{2m}{3} + \frac{1}{4}\right)$$

$$= \pi R^4\left(-\frac{m^4}{12} + \frac{m^2}{2} - \frac{2m}{3} + \frac{1}{4}\right)$$

Moment of Inertia About the Central Axis.



$$I_A = \frac{1}{2}\int r^2 dv$$

$$dv = \pi R^3(\cos\theta - m)^2 \cos\theta d\theta$$

$$r = R(\cos\theta - m)$$

$$m = D/R, \qquad \cos\phi = m, \qquad \sin\phi = a$$

$$I_A = \frac{\pi R^5}{2}\int_0^{\phi} (\cos\theta - m)^4 \cos\theta d\theta$$

$$= \frac{\pi R^5}{2}\int_0^{\phi}(\cos^4\theta - 4\cos^3\theta m + 6\cos\theta^2 m^2 - 4\cos\theta m^3 + m^4)\cos\theta d\theta$$

$$= \frac{\pi R^5}{2}\int_0^{\phi}(\cos^5\theta - 4m\cos^4\theta + 6m^2\cos^3\theta - 4m^3\cos^2\theta + m^4\cos\theta)$$

$$\int \cos^5\theta = \left[\frac{\cos^4\theta\sin\theta}{5} + \frac{4}{5}\left(\sin\theta - \frac{\sin^3\theta}{3}\right)\right]_0^{\phi}$$

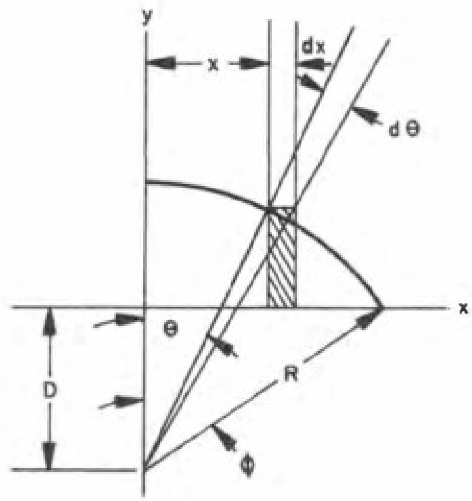
$$\int \cos^4\theta = \left[\frac{\cos^3\theta\sin\theta}{4} + \frac{3}{4}\left(\frac{\theta}{2} + \frac{\sin\theta\cos\theta}{2}\right)\right]_0^{\phi}$$

$$\int \cos^3 \theta = \left[\sin \theta - \frac{\sin^3 \theta}{3}\right]_0^{\phi}$$

$$\int \cos^2 \theta = \left[\frac{\theta}{2} + \frac{\sin \theta \cos \theta}{2}\right]_0^{\phi}$$

$$I_A = \frac{\pi R^5}{2}\left[-\frac{9m^4 a}{5} + \frac{9m^2 a}{2} - 2m^2 a^3 + \frac{4a}{5} - \frac{4a^3}{15} - \left(2m^3 + \frac{3m}{2}\right)\phi\right]$$

Moment of Inertia About the Base Plane.



dI_c = differential of inertia about the centroid

$$I_B = \int dI_c + \int x^2 dv$$

$$dI_c = \frac{dv}{12}\left[3y^2 + (dx)^2\right]$$

$$dv = \pi y^2 dx$$

$$dI_c = (y^2 dv/4)$$

$$m = D/R = \cos \phi$$

$$a = L/R = \sin \phi$$

$$I_B = \int \frac{y^2 \pi y^2 dx}{4} + \int x^2 \pi y^2 dx$$

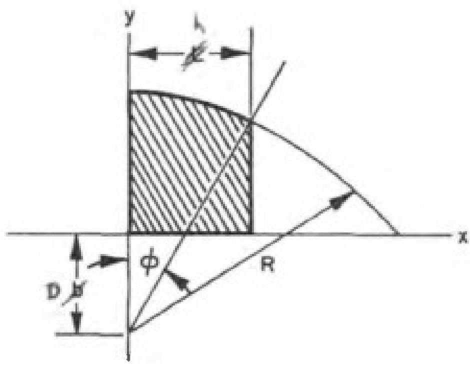
$$= \pi \int \left(\frac{y^4}{4} + y^2 x^2\right) dx$$

$$I_B = \pi R^5 \int \left[\frac{(\cos \theta - m)^4 \cos \theta d\theta}{4} + (\cos \theta - m)^2 \sin^2 \theta \cos \theta d\theta\right]$$

$$I_B = \pi R^5 \left[-\frac{9m^4a}{20} + \frac{9m^2a}{8} - \frac{m^2a^3}{2} + \frac{a}{5} - \frac{a^3}{15} - \frac{1}{4}\left(2m^3 + \frac{3m}{2}\right)\phi + \frac{3m^4a}{10} - \frac{m^2a}{4} + \frac{m^2a^3}{3} + \frac{a}{5} - \frac{a^3}{15} - \frac{m}{4}\phi \right]$$

$$= \pi R^5 \left[-\frac{3m^4a}{20} + \frac{7m^2a}{8} - \frac{m^2a^3}{6} + \frac{2a}{5} - \frac{2a^3}{15} - \left(\frac{m^3}{2} + \frac{5m}{8}\right)\phi \right]$$

Truncated Ogive



Moment.

$$M = \pi R^4 \left(-\frac{b^4}{4} + \frac{2mb^3}{3} - \frac{m^2b^2}{2} + \frac{m^2}{2} - \frac{2m}{3} + \frac{1}{4} \right)$$

Volume.

$$V = \pi R^3 \left(-\frac{a^3}{3} + a + m^2a - mab - m\phi \right)$$

Moment of Inertia About the Central Axis.

$$I_A = \frac{\pi R^5}{2} \left[\frac{b^4a}{5} - b^3ma - 2bm^3a - \frac{3bma}{2} - 2a^3m^2 - \frac{4a^3}{15} + \frac{4a}{5} + m^4a + 6am^2 - \left(2m^3 + \frac{3m}{2}\right)\phi \right]$$

Moment of Inertia About the Base Plane.

$$I_B = \pi R^5 \left[-\frac{3b^4a}{20} + \frac{mb^3a}{4} - \frac{5mab}{8} - \frac{m^3ab}{2} - \frac{2a^3}{15} - \frac{m^2a^3}{6} + \frac{m^4a}{4} + \frac{3m^2a}{2} + \frac{2a}{5} - \left(\frac{m^3}{2} + \frac{5m}{8}\right)\phi \right]$$

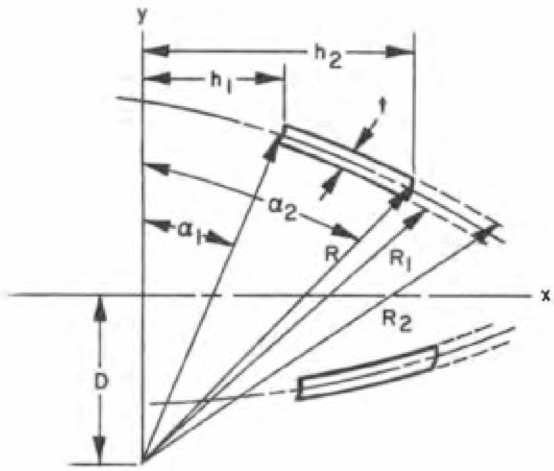
where

$a = \sout{L/R = \sin \phi}$ $a = h/R = \sin \phi$

$b = \sqrt{1 - a^2} = \cos \phi$

$m = D/R$

THIN-SHELLED OGIVE



The thin ogival shell illustrated is one commonly encountered in the design of a missile nose cone. Simplified approximations have been developed that offer the designer a short-cut method of determining the properties of this type of section. These relationships are

$$V = 2\pi R^2 t\left[(a_2 - a_1) - m(\alpha_2 - \alpha_1)\right]$$

$$M = 2\pi R^3 t\left[\frac{(a_2^2 - a_1^2)}{2} + m\left(\sqrt{1 - a_2^2} - \sqrt{1 - a_1^2}\right)\right]$$

$$I_A = 2\pi R^4 t\left\{(a_2 - a_1)(1 + 3m^2) - \frac{(a_2^3 - a_1^3)}{3} - m^3(\alpha_2 - \alpha_1)\right.$$

$$\left. - \frac{3m}{2}\left[(\alpha_2 - \alpha_1) + (a_2\sqrt{1 - a_2^2} - a_1\sqrt{1 - a_1^2})\right]\right\}$$

$$I_B = 2\pi R^4 t\left\{\frac{(a_2^3 - a_1^3)}{3} - \frac{m}{2}\left[(\alpha_2 - \alpha_1) - (a_2\sqrt{1 - a_2^2} - a_1\sqrt{1 - a_1^2})\right]\right\} + \frac{1}{2} I_A$$

where

$t = R_2 - R_1$

$$R = \frac{R_1 + R_2}{2}$$

$m = D/R$

$a = h/R$

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U. S. Naval Ordnance Test Station
Handbook of Equations for Mass and Area Properties of Various Geometrical Shapes, Compiled by Jack A. Myers. China Lake, Calif., NOTS, April 1962. 92 pp. (NAVWEPS Report 7827, NOTS TP 2838), UNCLASSIFIED.

ABSTRACT. This publication is a compilation of equations for moments of inertia, centroidal distances, radii of gyration, and other mathematical properties related to solids, thin shells, thin rods, plane areas, and ogival shapes.

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